



## Biometrics

# Accurate Estimation of Commercial Volume in Tropical Forests

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Accurate estimates of commercial volume in tropical forests are key for the implementation of sustainable forest management plans. Because of the lack of local or generic volumetric equations, most forest managers and forestry services are still using traditional expansion factors (i.e., multiplication of the diameter by a given value) to estimate the volume of commercial tree species in the Amazon. Volumetric models were developed through a unique data set of 1,264 fallen trees fully measured in 150 sample plots located across a broad range of forests in Amapá, Brazil. Forest-specific volumetric models were developed and compared with a generic (i.e., across all forests) model and with published equations developed elsewhere in the Amazon. The generic equation performed well in all forest types and allowed precise predictions. The most efficient sampling design to develop volumetric models consists of measuring approximately 50 trees across four different size classes representing the whole population. The form factors (FF) developed locally generated substantial bias but performed better than the traditional FF (0.7). Overall, our results suggest that it is possible to develop accurate generic models to estimate commercial timber volume, and this study can serve as a guideline for forest managers or scientists interested in calibrating volumetric models in a cost-efficient way.

**Study Implications:** This work provides useful information on volumetric modeling methods for Brazilian Amazon tropical forests. Most of the studies in the literature only investigate the classical modeling using regression models considering only boom metrics with or without bark, and, in this way, they provide incomplete and biased total knowledge and estimates for a given population. Therefore, detailed and accurate analyzes are crucial tools for decisionmaking. If the harvesting interventions are carried out without considering the most appropriate method to estimate the total wood stock, there may be damages or even extinction of some species, as has happened with other forest domains in Brazil and in other rainforest regions in the world. In this work, the results clearly show the importance of testing different methodologies and selecting the one best suited for a particular site, as well as carrying out techniques for the sustainable and correct management of the forest. Because the analysis procedures provide only information on how methodologies behave statistically, our results may contribute to a more refined analysis to be applied in the future in similar environments. Currently, the Brazilian forestry sector is looking for alternatives to obtain forest resources within the concept of sustainability. For the Brazilian Amazon tropical forest domain, it is extremely important to achieve a sustainable management of resources through forest management. Most studies in the literature investigate the management of tropical rainforest, whereas there is a lack of scientific information on the transition range for the cerrado.

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In the tropics, approximately 40 percent of the sawn wood traded annually comes from natural forests (Payn et al. 2015). Brazil is among the larger producers of tropical roundwood, with 81 million m<sup>3</sup> (48 percent of total production) of logs harvested annually (2005–2008) in natural tropical forests (Blaser and International Tropical Timber Organization 2011, da Silva et al. 2018). In its efforts to promote sustainable forest management, Brazil's government has enforced a set of regulations, defining notably a maximum allowable harvest of 30 m<sup>3</sup> per hectare when mechanized and 10 m<sup>3</sup> per hectare when operations do not involve heavy machinery, over a 35- and 10-year cutting cycle, respectively (Roma 2013). Accurate estimation of commercial volume at both tree and stand level is thus key to efficiently implementing such regulations and ensuring sustainable management of forest resources (Buongiorno and Gilless 2003, Burkhardt and Tomé 2012). In a broader context, commercial timber volume (CTV) provides a rapid and easy way to estimate the monetary value of trees or forest stands, often referred to as commercial timber stock (Bettinger 2009).

In the field, CTV is generally estimated for the main trunk, as the multiplication of the diameter at breast height (DBH; 130 cm), the commercial height (Hc) of the trunk, and a form factor (FF).

Hc is generally defined as the lowest main branch forming the base of the crown (Ploton et al. 2016, Rutishauser et al. 2016), and FF represents the ratio between the bole volume and that of a cylinder of the same girth and height (Gray 1956, Burkhardt and Tomé 2012). FFs were shown to perform well when applied to a few commercial species, or when developed and used locally (Gray 1956, Colpini et al. 2009), and most forest services in the Brazilian Amazon are commonly applying a FF of 0.7 (Heinsdijk and Bastos 1963, Colpini et al. 2009). Indeed, volume, height, or shape of trees are known to greatly vary among and within species (Cannell 1984, Akindele and LeMay 2006), across large ecological gradients (Nogueira et al. 2008, Lines et al. 2012), or at smaller scale due to competition for light and nutrients (Iida et al. 2011, Rutishauser et al. 2016). To account for this variability, various volumetric equations have been developed across the Amazon (Rolim et al. 2006, da Silva and Santana 2014), and the use of a single FF at regional scale remains to be evaluated.

Recently, the state of Amapá (Figure 1) declared 2.3 million hectares (16.5 percent of its area) of natural old-growth forests as state forests to tackle rapid deforestation and forest degradation and promote a sustainable timber harvest (Instituto Estadual de

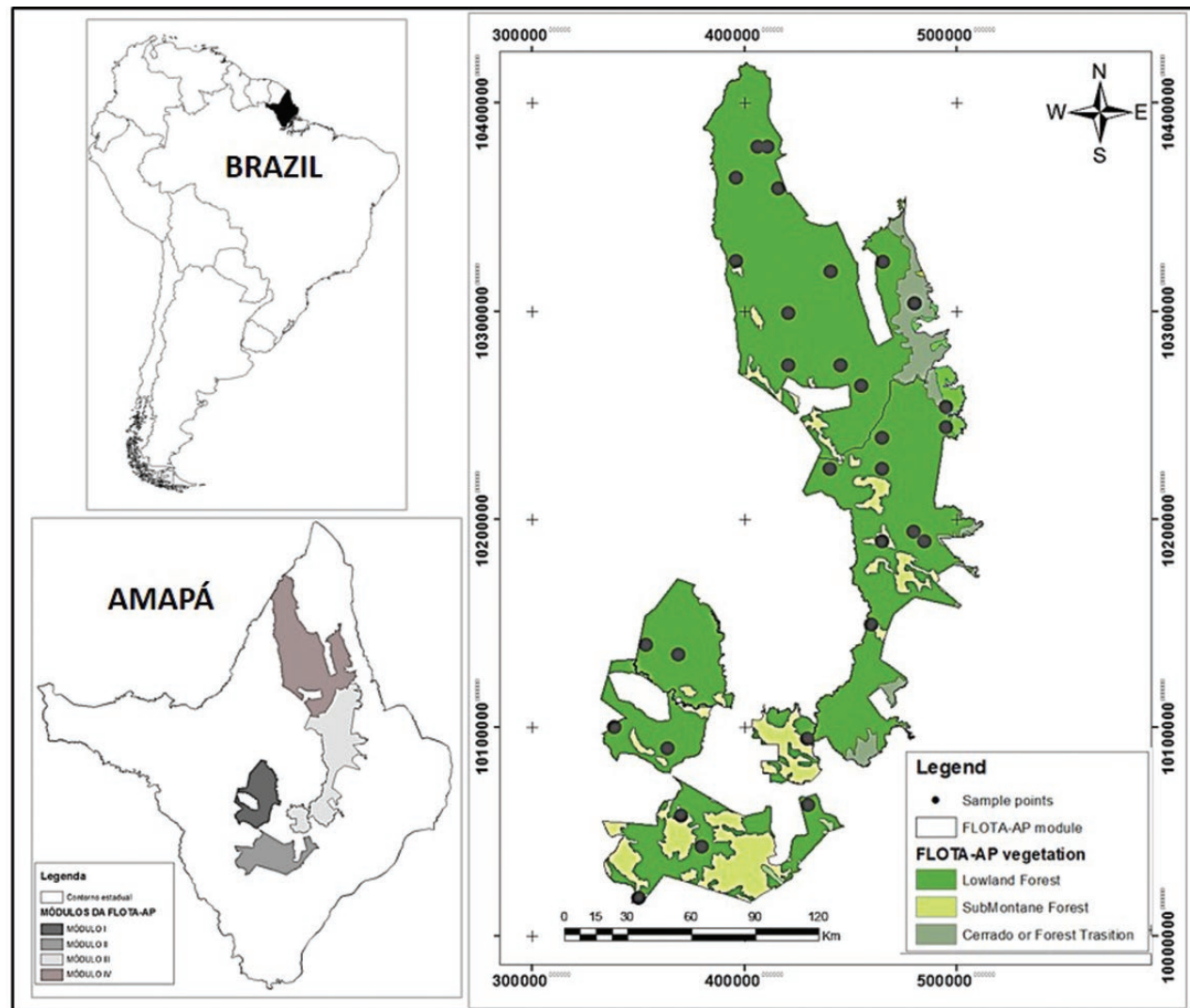


Figure 1. Location of state forests in Amapá, Brazil (left panel). Sample units (points) were randomly located in the different forest types, after stratification in function of the area of each forest type (right panel).



To select the best model, the goodness of fit was compared through Akaike information criterion (AIC) coefficient of determination, root-mean-square error (RMSE), and bias (see [Supplementary Materials](#)). Once the best model was selected, we sequentially tested for a “forest type” effect on each model parameter adding a random “forest type” variable in place of the tested parameters. Using a “forward stepwise” selection based on AIC, we built the final model. Because of the difficulty of testing for a forest type effect on several parameters at once, the effect was tested parameter by parameter. A significant effect was detected if the likelihood improved  $>\log(\#trees)$  compared with the null model (without forest type; e.g., [Vibrans et al. 2015](#)). The same procedure is repeated for the remaining parameters. This ad hoc procedure is similar to a Bayesian information criterion forward procedure in a `glm()` framework.

### Minimum Sample Size to Develop Accurate Volumetric Models

To provide some guidance regarding the development of volumetric models, we tested two sampling methods (simple random sampling and random sampling within the strata) and various sample sizes to propose an optimal strategy. We simulated a random sampling strategy consisting of randomly selecting N individuals among either all trees  $DBH \geq 10$  cm or only commercial trees (i.e.,  $DBH \geq 50$  cm).

Appropriately, 2, 5, 10, 20, 50, and 70 percent of either all trees  $DBH \geq 10$  cm or all trees  $DBH \geq 50$  cm were selected randomly. Alternatively, we simulated a stratified random sampling in grouping trees into four size classes of equal strata size (5–20, 20–35, 35–50, and  $>50$  cm for all trees  $DBH > 10$  cm and 50–55, 56–65, 66–75, and  $>75$  cm for commercial trees). Similarly 2, 5, 10, 20, 50, and 70 percent of trees within a given class were randomly chosen.

**Table 2. Published equations used for comparison in the prediction of CTV. All models are of the form:  $\log(CTV) = \beta_0 + \beta_1 \cdot \log(DBH) + \beta_2 \cdot \log(Hc) + \varepsilon$ .**

$CTV_{ALT}$	$\beta_0$	$\beta_1$	$\beta_2$	$R^2$	RSE	Ntrees
<a href="#">Thaines et al. (2010)</a> : Amazonas	-9.5452	2.1284	0.7221	0.92	0.7048	141
<a href="#">Silva and Santana (2014)</a> : Pará	-9.5084	2.0139	0.8788	0.94	0.1103	234
<a href="#">Silva et al. (1984)</a> : Pará	-8.8610	1.9318	0.7868	0.96	0.6294	905

Note:  $\beta_i$ , parameters to be estimated; CTV, commercial timber volume above bark ( $m^3$ ); DBH, diameter at breast height (cm);  $\varepsilon$ , random error; Hc, commercial height (m); Log, logarithm natural; Ntrees, number of trees used in the study cited; RSE, residual standard error.

**Table 3. Parameter estimates (fitted in a hierarchical framework) and indices of goodness-of-fit of four volumetric models across all and by forest types (lowland forests; submontane forests; cerrado/forest transition; [Table S1](#)). The best model is bolded.**

Area	N	Model	$\hat{\beta}_1 \pm SE$	$\hat{\beta}_2 \pm SE$	$\hat{\beta}_3 \pm SE$	AIC	RSE	$R^2_{adj}$	RMSE	Bias (%)
All forest types	1,264	Husch	-8.031 ± 0.081	2.210 ± 0.023		1471.87	0.43	0.88	0.8	20
		Hohenald-Krenn	-7.508 ± 0.384	1.898 ± 0.225	0.045 ± 0.033	1471.93	0.43	0.88	0.82	19
		Spurr	-8.899 ± 0.054	0.925 ± 0.006		263.87	0.27	0.95	0.49	7
		Schumacher-Hall	-8.889 ± 0.054	1.881 ± 0.016	0.875 ± 0.019	258.51	<b>0.26</b>	<b>0.96</b>	<b>0.48</b>	<b>6</b>

Note: Corresponds to the statistical parameters of the models and the confidence interval measured by the standard error of the estimate. AIC, Akaike information criterion;  $R^2_{adj}$ , adjusted coefficient of determination; RMSE, root-mean-square error; RSE, residual standard error.

For each subset, the best local model was developed and applied to the remaining data for cross validation. For each iteration, RMSE and bias were computed, and this procedure was carried out 1,000 times for each sample size. We report mean RMSE and bias with 95 percent confidence interval for both sampling strategies and population of trees (e.g., [Sullivan et al. 2018](#)).

### Form Factors

FFs were computed at tree level, as the ratio between estimated CTV and that of a cylinder of diameter equal to DBH and height equal to Hc. FFs were developed for every tree and averaged by forest type and among all forests ( $FF_{ALL}$ ). We also tested the performance of the traditional FF ( $FF = 0.7$ ).

### Applicability of Published Volumetric Models

Three alternative volumetric models ( $CTV_{ALT}$ ) developed in other Amazonian regions were compared with measured CTV ( $CTV_{MES}$ ) ([Table 2](#)).

For each equation, standard error (RSE) and coefficient of variation (CV;  $p < .05$ ) were computed for each forest type ( $j$ ) as follows:

$$RSE(j) = \sqrt{\frac{1}{N_j - p} \sum_{i=1}^N [CTV_{ALTij} - CTV_{MESij}]^2} \quad (2)$$

$$MCTV(j) = \frac{1}{N_j} \sum_i CTV_{MESij} \quad (3)$$

$$CV(j) \% = \frac{RSE(j)}{MCTV(j)} \times 100 \quad (4)$$

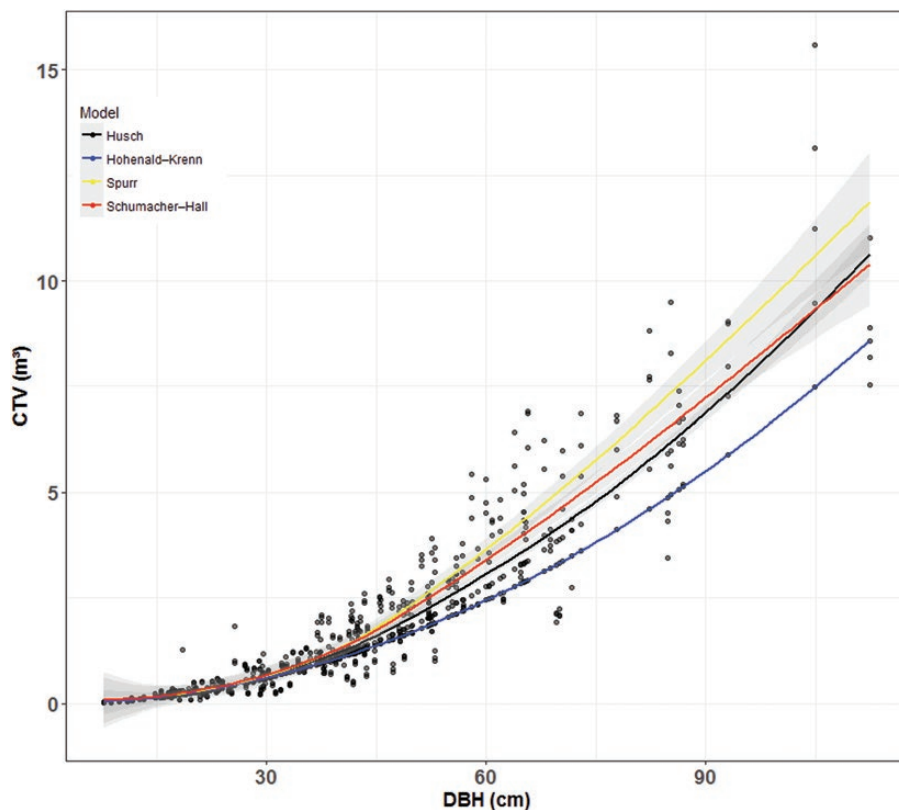
where  $CTV_{ALTij}$  and  $CTV_{MESij}$  are the volume estimates of tree  $i$  in forest type  $j$ . A large value of CV would be acceptable as long as the bias is low because, in general, the model is applied among many trees within a site, and therefore, random errors tend to cancel out ([Chave et al. 2014](#)). All computations and analyzes were performed using the statistical software R ([R Development Core Team 2017](#)).

## Results

### Development of Volumetric Models

Among the four adjusted volumetric models, models that include DBH and Hc (i.e., Schumacher-Hall and Spurr) presented better performance (lower AIC; [Table 3](#)), returned the best predictions, and explained more than 95 percent of the total variance ( $R^2_{adj}$ ; [Table 3](#)), despite large variations in Hc for a given DBH ([Figures S1–S5](#)).

Goodness of fit (RMSE) and prediction error (bias) roughly doubled when estimating CTV through DBH only ([Table 3](#)). Generally, CTV of trees  $DBH < 50$  cm were well predicted by all models but tended to diverge for larger diameters ([Figure 3](#)). The



**Figure 3.** Regression curves (colored lines) and 95 percent confidence intervals (gray shading) of four models (detailed in Table 1) relating commercial timber volume (CTV) and diameter at breast height (DBH) for overall forest types (“all forests types”).

Schumacher-Hall model was found to return the best estimates in both Sub and Trans forests (Table S1 and Figure S6, and across all forest types (Table 3). This model generally captured variations well in DBH and Hc, resulting in a mean bias of only 6 percent (Table 3). No significant effect of forest type on the estimated parameters was detected (Table 4), pointing out the genericity of this model at regional scale.

#### Minimum Sample Size

The model parametrization carried out above already revealed that stratification by forest type was unnecessary. Regarding the minimum sample size needed to develop such an accurate generic model, all strategies tested tended to plateau above 20 percent (approximately 250 trees) of the whole population sampled (Figure 4a). Although the error on individual CTV estimates was on average higher when models were developed solely with trees DBH > 50 cm (Figure 4a), errors tended to cancel out at stand level when large trees were sampled in a stratified manner (Figure 4b). Overall, the best option to estimate CTV is sampling approximately 50 trees across four different size classes representing the full DBH ( $\geq 10$  cm) distribution. However, results from simulations show that a stratified sampling among large commercial trees returns fair stand-level CTV estimates (<5 percent difference).

#### Comparing Published Volumetric Equations and Form Factors

Predictions of the model proposed by Silva et al. (1984) were similar with those of our best model (Figure 5). The average CV was 32.2 percent and 31.9 percent using the best local model and Silva’s model, respectively (Table 5). The regional models of Thaines

**Table 4.** Parameter estimates of the Schumacher-Hall model [ $CTV \sim \log(\beta_0 \cdot DBH^{\beta_1} \cdot Hc^{\beta_2}, \epsilon)$ ], when including forest types (Low, lowland forests; Sub, submontane forests; Trans, cerrado/forest transition). RMSE and likelihood of the whole model. Note that the null-model (i.e., without forest type) likelihood is 1,029.8.

	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$
Low	0.786	1.872	0.878
Sub	0.779	1.904	0.864
Trans	0.786	1.900	0.867
RMSE	0.267	0.267	0.267
Likelihood	1,029.6	1,025.5	1,029.5

Note: Corresponds to the statistical parameters of the models obtained by generalized linear modeling.  $\beta_i$ , parameters to be estimated; CTVlog, logarithm of the commercial timber volume; DBH, diameter at breast height (cm);  $\epsilon$ , random error; Hc, commercial height (m); RMSE, root-mean-square error.

et al. (2010) and da Silva and Santana (2014) underestimated CTVs in smaller trees across all forest types (Figure 5). Yet, this trend remained when considering the forest types independently, with greater evidence in submontane forests and forest transition (Figure S7).

FFs were generally close to 0.87 in all forest types but overestimated actual CTV by 38 percent (Figure 5 and Table 5). CTV predictions through FFs showed greater variation and errors in all forest types (Table S2 and Figure S7).

## Discussion

### Accurate Estimation of CTV in Northern Amazonian Forests

The present study investigates the ability of a newly developed generic volumetric model, along with three published ones,



equations developed locally (Conselho Nacional do Meio Ambiente [CONAMA] 2009). However, no recommendation is made about the minimal size or design (i.e., random versus stratified) of the sample, impeding efficient implementation of this regulation.

Although total height measurements are often time consuming and sometimes hard to achieve in tall tropical forests (Sullivan et al. 2018), measuring trunk height is less difficult and is often routinely done by forestry services. However, instead of measuring the trunk height of all trees, we suggest measuring a stratified sample of the whole stand. Our results tend to show that models developed through a stratified random sample are generally more accurate than those based on simple random samples, when all trees DBH > 10 cm are considered. The likely reason is that random sampling skews sampling toward small stems, resulting in larger errors across larger trees that are under-represented (Duncanson et al. 2015). Stratified sampling lessens CTV errors and tending to zero, even when a small fraction of the whole population is measured (Figure 4b). Subsequently, our results suggest that choosing 50 trees across the different size classes (stratified sampling) is the most efficient way to develop a generic volumetric model.

The inclusion of total height was shown to improve whole live-tree biomass (Chave et al. 2005); our results confirm that trunk heights vary across forests and should be systematically measured. Accounting for both DBH and Hc captured the variability among plots and forest types, up to the point that a single generic model performs better than forest-specific models. Interestingly, the parameters related to trunk volume (i.e.,  $\beta_1$  and  $\beta_2$ ) of our generic model were very close to other models developed in Amazon forests (Thaines et al. 2010, da Silva and Santana 2014). This indicates that, contrary to trunk height, trunk volume does not vary greatly at regional scale and that our generic model could be applied elsewhere in the Amazon basin.

## Conclusion

Accurate volume estimates can be obtained by the Schumacher-Hall equation for the forest in Amapá. Estimates with valid confidence intervals can be obtained using only a generic equation without the need to include forest type.

The FFs produced less reliable estimates than the local equation and suggest larger prediction errors when used for large scales (live-tree inventory), where specific volume estimates are required.

Our study pointed out that, contrary to what is generally thought, generic models to estimate CTV can be developed. Integrating volumetric data at the scale of the whole Amazon basin could potentially lead to the development of a single model integrating, for instance, DBH, Hc, and key climatic information. Such a generic model could then be used reliably by the forestry sector to enforce a sustainable forest management of the remaining natural forests in the region.

## Supplementary Materials

Supplementary data are available at *Forest Science* online.

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