Planning of production units for native forest management areas in the Amazon

Evandro Ferreira da Silva¹, Rodrigo Freitas Silva¹, Giovanni Correia Vieira¹, Catherine Cristina Claros Leite¹, Marcelo Otone Aguiar¹, Evandro Orfanó Figueiredo², Mayra Luiza Marques da Silva³, Gilson Fernandes da Silva³

¹ Universidade Federal do Espírito Santo/Centro de Ciências Agrárias/Departamento de Engenharia Florestal. Jerônimo Monteiro, ES, Brasil. E-mail: evandroflorestal@gmail.com (ORCID: 0000-0001-5948-6402); rodrigo_freitas_silva@yahoo.com.br (ORCID: 0000-0002-2734-5370); ogiovanni@gmail.com (ORCID: 0000-0003-4892-9241); catherinecris@gmail.com (ORCID: 0000-0001-7853-6284)
² Embrapa Agroflorestal. Rio Branco, AC, Brasil. E-mail: evandro.figueiredo@embrapa.br (ORCID: 0000-0002-8181-7906)
³ Universidade Federal de São João Del-Rei/Departamento de Ciências Agrárias/Campus Sete Lagoas. Sete Lagoas, MG, Brasil. E-mail: mayralmsilva@gmail.com (ORCID: 0000-0001-9209-446X)

ABSTRACT: The objectives of this study were to present a model of integer linear programming (IP) to develop the creation of production units (PUs) regulated at the level of exploitable trees and to implement a heuristic method to obtain suboptimal feasible solutions. The studied area is in the municipality of Bujari, AC, Brazil. Using a sustainable forest management (SFM) area of 1,057.41 ha, 4,237 trees were selected for exploration through the census inventory. We applied the $p$-median model with volume and income restrictions; however, the adaptive heuristics development (AHD) is a random, greedy, and adaptive procedure that forms the PUs optimally. Four scenarios were formulated with a variation of ±10% to ±20% in volume and income values; the simulation was repeated 10 times with 10,000 iterations in each scenario. As expected, the major variations were in the scenarios with the most flexible restrictions of ±20%. Overall, 400,000 iterations were performed with average processing time of 523.74 s for each scenario. The methodology fostered the creation of PUs efficiently by grouping the trees into an optimized set, and at the same time, respecting the constraints of the problem.

Key words: annual unit production; optimization of forest harvesting; precision forestry management; sustainable forest management

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RESUMO: Os objetivos do estudo foram apresentar um modelo de programação linear inteira (PLI) para realizar a formação de unidades de produção (UP) reguladas ao nível de árvores exploráveis e implementar um método heurístico para obtenção de soluções viáveis subótimas. A área de estudo situa-se no município de Bujari, AC, Brasil. É uma área de manejo florestal sustentável (AMFS) com 1,057,41 ha. Por meio do inventário censitário foram selecionadas 4,237 árvores a serem exploradas. O modelo empregado foi o modelo das $p$-medianas com restrições de volume e renda, já a heurística de formação adaptativa (HFA) constitui-se de um procedimento aleatório, guloso e adaptativo para formar as UPs de maneira otimizada. Foram formulados quatro cenários com variação de ±10% e ±20% nos valores de volume e renda, sendo simulado dez vezes com 10,000 iterações cada cenário. Como esperado, as maiores variações foram nos cenários com flexibilização da restrição em ±20%. Ao todo, foram realizadas 400,000 iterações de soluções, com tempo médio de processamento 523,74 s para cada cenário. A metodologia promoveu a formação de UPs de forma eficiente, agrupando as árvores de maneira otimizada e, ao mesmo tempo, respeitando as restrições do problema.

Palavras-chave: unidade de produção anual; otimização da colheita florestal; manejo florestal de precisão; manejo florestal sustentável
Introduction

One of the main challenges in planning the exploitation of native forests is the regulation of forest production (Carvalho et al., 2015). Traditionally, the partition of a forestry management area (FMA) is into production units (PUs) of similar size and shape whenever possible, but this procedure does not necessarily correspond to a regular harvesting volume and income across each PU. Thus, the spatial heterogeneity of the distribution of species promotes oscillations in each area. When considering the planning, the PUs should be set as an annual production unit (APU), which are the same as PUs but defined for exploitation every year. Thus, there is a need to consider legal, economic, environmental, and technical restrictions.

The physical allocation of APUs can be set according to the census inventory, identification of productive areas, and areas of permanent preservation (APP), for example. The technical aspects are also important, such as those that define the intensity of the exploitation, exploitable individuals, allocation of storage yards, and the cutting cycle. Moreover, it can be considered market information regarding the commercial value of the species, distance to the consumer market, required feedstock, production costs, availability of labor, and technology to perform the exploitation of the forest.

The need for a detailed spatial planning has been notorious in recent years owing to economic, environmental, and production goals (Baskent & Keles, 2005). The spatial distribution of the PUs is a problem that requires optimized solutions that employ mathematical programming (MP) techniques (Souza & Soares, 2013). The application of the operational research (OP) has presented relevant contributions in forest planning (Baskent & Keles, 2005; Mathey et al., 2008; Fotakis, 2015; Liu & Lin, 2015).

The employment of this type of modeling in forest planning in native forests has been addressed in some studies (Braz et al., 2004; Fernandes et al., 2013; Martinhago, 2012; Silva, 2014, 2015). OP techniques represent a potential tool for solving problems in areas of sustainable forest management (SFM) in the Amazon.

The classic model of integer linear programming (IP) of \( p \)-medians consists of finding \( p \) (medians) in a graph in such a way as to minimize the sum of distances to each client (vertex) to the nearest median (Beasley, 1985; Avella et al., 2012; Fávero & Belfiore, 2013) resulting in the formation of clusters (Mladenović et al., 2007; Avella et al., 2012). Thus, the formation of PUs can be performed through clusters and governed by the restrictions imposed on the problem.

The \( p \)-median problem is recognized as an NP-hard problem (Colmenar et al., 2016); therefore, alternative strategies for the solution of the model are necessary. Commonly employed procedures based on heuristics and metaheuristics (Church & Beamr, 2008; Domínguez & Muñoz, 2008) are more efficient in obtaining viable solutions for IP. The heuristic method does not guarantee an exact solution; however, it is able to provide a viable solution that is at least close to the optimal, using a reasonable computational effort (Hillier & Lieberman, 2006).

In this work, we do not address the problem of the formation of APUs, because to do so would mean that the economic aspects and the growth of the forest would need to be considered in the model, according to the production cycle. The formation of PUs by this mathematical approach can be seen as a methodology for the development of regulated areas that can be negotiated in SFM plans with interested companies. The planning involves only short-term exploitation so that we can disregard the economic variation in time and volume increment in the forest, planning the scaling of the harvest, planning of costs and incomes estimated at harvest, among others.

The objective of this study was to present a model of IP to fulfill the development of PUs to the level of exploitable trees, promoting the regulation of forest production, and to implement a heuristic method for obtaining feasible suboptimal solutions.

Material and Methods

Area of study

The study area is in the municipality of Bujari, State of Acre, Brazil. It consists of a forest block of 1,057.41 ha of native rainforest, located at 593,626.26 m W and 8,957,685.44 m S (Figure 1).

Figure 1. Location and delimitation of the study area.

In the region, there are predominantly two types of forests, dense ombrophilous forest and open ombrophilous forest, both with a great diversity of plant formations. The climate of the region is of type Am, hot, and humid equatorial, with average annual temperature around 24.5°C (Governo do Estado do Acre, 2010).

Data processing

The data processing was split into steps, executed sequentially (Figure 2).
Step 1. Forest inventory census database

Through the forest inventory census conducted in the study area, we obtained the information of all the trees with a diameter at breast height (DBH) ≥ 30.0 cm. The variables measured were DBH (cm), coordinates of each tree, common and scientific name, commercial height (CH) (m), and quality of stem (1, good; 2, regular; 3, bad).

The trees were classified according to normative instruction nº 5, of December 11, 2006 of Ministry of Environment as: rare trees or prohibited to cut, seed-tree, future cut trees, trees of second cycle, trees located in APP, and exploitable trees. During the study, only the harvested trees were selected and the individual volume was obtained, which resulted in 4,237 individuals, with a total estimated volume of 25,437.45 m³, average volume of 6.0 m³ per tree, and the intensity of exploitation was 29.09 m³ ha⁻¹.

Step 2. Model input variables: location, volume, and income

The input data necessary for the optimization model were location (metric coordinate), volume (m³) that was obtained in Step 1, and income from wood (R$). Timber prices (R$ m⁻³ in timber) were obtained from a bulletin of agricultural and forestry products prices lifted by the Brazilian Agricultural Research Company of Acre (Embrapa Acre) and the Federation of Agriculture and Livestock of the State of Acre (FAEAC) in 2013 (Embrapa Acre; FAEAC, 2013). Thus, the individual income generated by each of the exploitable trees has been calculated by multiplying the volume by the price of a cubic meter of the species at the local market.

Step 3. Euclidean distance matrix of each tree location

In addition to the variables in the previous step, other input necessary to the optimization model was the Euclidean distance, in meters, of every tree related to other trees in the area \(d_{xy}\), obtaining a symmetric matrix of dimension \(n\times n\),

\[
d_{xy} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]  

Step 4. Mathematical model and heuristic adaptive development

The mathematical model was proposed based on the model of the \(p\)-median (Fávero & Belfiore, 2013; Goldbarg, 2015) to group individuals of exploitable commercial value to form PUs and performing the adjustment of forestry production volume and income restrictions:

Minimize : \[
\sum_{i=1}^{p} \sum_{j=1}^{n} d_{ij} x_{ij}
\]  

Subject to:

\[
\sum_{j=1}^{p} x_{ij} + 1
\]  

\[
\sum_{j=1}^{p} Y_j = P
\]  

\[
x_{ij} - Y_j \leq 0
\]  

\[
\sum_{i=1}^{n} R x_{ij} \leq \frac{R}{P} (1 + I_j)
\]  

\[
\sum_{i=1}^{n} R x_{ij} \geq \frac{R}{P} (1 + I_j) y_j
\]  

\[
\sum_{i=1}^{n} V x_{ij} \leq \frac{V}{P} (1 + I_j)
\]  

\[
\sum_{i=1}^{n} V x_{ij} \geq \frac{V}{P} (1 - I_j) y_j
\]  

where \(i\) is the \(i\)th exploitable tree, \(j\) is the central tree PU, \(d_{ij}\) is the Euclidean distance from the tree \(i\) to the tree \(j\) in the PU center, \(P\) is the total number of PUs to be formed, \(x_{ij}\) is a binary variable that takes the value 1 if the tree \(i\) is connected to the tree \(j\) in the center of the tree PU and 0 otherwise, \(Y_j\) is a binary variable that takes the value 1 if the tree \(j\) is in the center tree of the PU and 0 otherwise, \(R\) is the
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The total income of the SFM area, \( r_i \), is the income of the \( i \)th tree, \( v_i \) is the volume of the \( i \)th tree, \( v \) is the total volume of the SFM area, \( I_r \), is the rate of change allowed for income, and \( I_v \) is the rate of change allowed for volume.

The function objective (2) seeks to minimize the sum of the Euclidean distances between each tree and the tree selected to be the center of the cluster formation of each PU. The first restriction (3) ensures that each tree is connected only to one center tree of all the PUs formed, i.e., each tree will belong exclusively to one PU. The constraint (4) establishes the number of PUs to be formed in the area of SFM. The constraint (5) ensures that each tree is connected only to a tree that is selected as the center tree. The restrictions (6) and (7) regulate the income of each PU in line with the rate of variation allowed. The restrictions (8) and (9) regulate the volume of each PU according to the rate of variation allowed. The last restriction (10) ensures that the decision variables are binary.

Step 5. Adaptive heuristic development (AHD)

To resolve the model, a heuristic algorithm was developed and the flowchart is presented in Figure 3. The heuristic method proposed consists of a random, greedy, and adaptive procedure to optimally form the PUs. Its implementation is divided into two parts and performed consecutively.

First, respecting the amount of established PU, randomly choose between all the eligible geographical points (trees) those that have become central point of a PU, that is, the central tree of each PU. The locations are taken from the coordinates of exploitable trees, inventoried in the SFM area, and entered as the input parameter of the problem.

Second, the PUs are set by groups of exploitable trees, the shortest distance between them, and the location of their center points selected (tree centers). During this moment of heuristic solution, the viability of the solution is verified according to the pre-established regulation for volume and income. If it is not a viable solution, iteratively changes are

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**Figure 3.** Flowchart of the heuristic proposal.
made in the groupings in order to allow the solution to adapt to the constraints of the model.

Thus, in resolution, individuals are grouped by the greedy part of the PUs where the distance found between the trees and the center point (central tree) of the PUs is the smallest possible. The greedy strategy (Cormen et al., 2012) is used in this method as the decisions regarding the development of PUs are better at first, given the information available. However, it will not analyze the consequences of those decisions in the final solution of the problem (Toscani & Veloso, 2012). It is important to highlight that greedy algorithms have been applied to forest resource optimization problems by Silva et al. (2015) and Resende & Werneck (2004).

Initially in the algorithm, in the grouping of trees and formation of PUs, there is no concern about the regulation of forest areas. The only goal is to find a global solution that minimizes the average distance of the trees to the center point of the PUs. Thus, it is important to note that there is a high probability that the generated solution at this stage will not be valid for the model constraints regarding the volume and income limits.

The solution obtained is initially evaluated in the second phase of implementation of the AHD. It will be considered viable if all the restrictions imposed by the mathematical model are satisfied and no viable case violates one or more of the constraints. If the solution is viable, the other solutions generated by the heuristic on previous executions are verified to check whether this is the best solution to the problem. If it is, the solution is stored. Otherwise, it is discarded. The best solution is the one that minimizes the objective function of the mathematical model; that is, the one that has the smallest average distance between each tree and the center tree of the PU.

However, if the output of the first stage of the AHD is a non-viable solution, this will undergo changes to adapt to the constraints of the problem until it becomes a viable solution. These adjustments refer to a rearrangement in the formation of PUs, although all of them remain with the same center points. The goal is to move the trees farther apart from a PU that has excess volume production of wood, allocating it to another PU next to it that has not yet reached the minimum volumetric production limit. This is similar for the total income. This procedure is then executed repeatedly until all the PUs fit all the limits and conditions established.

It may happen, however, that no group exceeds the production ceilings, although one or more may not have reached some minimum limit. In this case, the rearrangement occurs to insert the trees closest to the PU deficit and in other formations, as long as the PUs, which are the trees, remain within the constraints of the problem. The trees relocated are those that are on the border between two PUs. Finally, to produce a viable solution, this is compared with the others already generated by checking whether it was the best result found. If so, the solution is stored. Otherwise, the solution is simply discarded.

The stop condition of the AHD is the number of times that this method should be executed, each time generating a viable and different solution. After reaching the stop condition, the proposed heuristic presents as output the best solution found. The stop criterion was considered as the generation of 10,000 solutions for each scenario.

Step 6. Scenarios for the formation of PUs

Four scenarios were analyzed; the income and volume had a range from ±10% to ±20%.
- Scenario 1: income ±10% and volume ±10%.
- Scenario 2: income range ±10% and volume ±20%.
- Scenario 3: income ±20% and volume ±10%.
- Scenario 4: income and variable volume ±20%.

Step 7. Evaluation of the heuristic solution

To evaluate the method of solution of the proposed model, 10 simulations were performed for each scenario seeking in each 10,000 simulated solutions, with the aim of assessing the stability of the proposed heuristic in obtaining viable solutions. Finally, the descriptive statistics of the variables were calculated for each scenario: the average and coefficient of variation (CV) and a graphical presentation of a viable solution generated for each scenario.

Results and Discussion

The formation of PUs in SFM appears as the basis of planning, traditionally being realized empirically. The mathematical model and the AHF applied in four scenarios analyzed proposed the formation of areas governed by the restrictions of volume and income. The variations proposed in the scenarios are based on field knowledge, professional information, and management area exploration reports. A rate of volume change (m³) and income (R$) was applied, aiming to promote a loosening of restrictions to search for the viable solution, and that is consistent with the planning adopted by companies, in order to foster the tree clustering, resulting in the formation of PUs that conform with the restrictions.

The PUs in management areas are defined by estimating the production per unit area (m³ ha⁻¹) times the amount of area needed to obtain a desired production; in many cases, this oscillates above 50% of the expected volume. This oscillation interferes directly in the sizing of the exploration team, variable costs, and length of exploration time.

In addition, this flexibility applied to set maximum and minimum limits to the restrictions of the model can be adjusted according to the needs of the decision maker, configuring the model application to everyday situations in the decision-making process of forest planning.

In Figure 4, the best solution found for each of the scenarios assessed after 10 repetitions with 10,000 simulated
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The formation of PUs was represented by different colors with each point representing a tree inventoried. The method of resolution aimed at the formation of PUs, grouping the trees optimally, and at the same time, respecting the constraints of the problem.

The viability of the model can be seen in Figures 5 and 6: all restrictions were respected in the scenarios assessed, having a regular distribution between the limits.

The CV was calculated as it measures the dispersion of relative variability in the average function, and consequently measured whether the values obtained as a solution in 10 of each scenario simulations have been scattered according to the average and thus infer based on the resolutions obtained about the ability of the heuristics in obtaining viable resolution.

Thus, as expected, the scenarios with a higher percentage of softening (20%) achieved the largest ± CV.

The values obtained in Table 1 demonstrate the efficiency of the model in regulating production. Ebata Produtos Florestais (2017) made available its sustainable forest management plan (PMFS) for 2014 available, which mentions the presence of natural divisors as a criterion for the delimitation of APUs in the field. The PMFS presented the expected yield in each APU, with the APU size being 500 to 680.72 ha, with a cut intensity of 20–25.8 m³ ha⁻¹, and wood production of 10,000–17,555 m³ year⁻¹. In the annual operating plan for 2015, the APU was 700 ha, with a rectangular format, a cut intensity of 23.57 m³ ha⁻¹, and total volume of 15,114.44 m³ year⁻¹.
Thus, in this practical example, the criteria for the definition of PUs can vary according to the criteria of the decision maker, can be performed subjectively with low precision of the expected yields. In the case presented previously, the annual production was met, but based on a range greater than 50% of the lower limit of volume production of 10,000 m³, without any guarantee of the knowledge for future PU yields. It is of extreme importance that we research techniques aimed at the planning of regulations in areas of SFM to ensure the regulation of production as a function of the variables of interest such as income, volume of a certain class of individuals, and size of teams.

We carried out 400,000 iterations, with average run times of 478.50, 516.89, 471.78, and 627.80 s for Scenarios 1–4, respectively. The average processing time of the scenarios was 523.74 s, which is acceptable in the search for the best solution. The stopping criterion adopted for 10,000 iterations was considered satisfactory to achieve viable solutions in accordance with the restrictions imposed (Figure 7).

The formation of PUs in an optimized form taking other variables into account must be tested. To save time, the MP can be used for troubleshooting when the number of combinations to be evaluated is small, and depending on the solution method, returns an exact resolution to the problem. However, currently, facing the continuous exploration of growing areas, for example, forest concession areas that have continuous production, and consequently a large number of possible distinct formations for the PUs to be implemented, it is crucial that studies be formulated to improve the planning activities of the SFM.

It is important that the search for alternative means to overcome the computational limitations that may occur during the resolution of problems of IP formulations of exact shape should be explored, even if based on heuristics that generate suboptimal resolutions, present viable resolutions to the exact value, and provide acceptable processing time.

**Conclusions**

Using mathematical programming, we explored scenarios consistent with the reality and complexity of the problem of developing the creation of PUs regulated at the level of exploitable trees in order to obtain results of the scenarios analyzed as a function of the proposed solution method. The results showed that the AHD was effective in obtaining solutions to the problem presented and could assist forest managers in decision-making to form PUs that handle the restrictions presented.

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**Table 1. Volume and income for each scenario evaluated.**

<table>
<thead>
<tr>
<th>PU</th>
<th>Volume (m³)</th>
<th>Income ± 10</th>
<th>CV%</th>
<th>Income ± 20</th>
<th>CV%</th>
</tr>
</thead>
<tbody>
<tr>
<td>± 10</td>
<td>1</td>
<td>1,149,877.07</td>
<td>5.66</td>
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</tr>
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<td></td>
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<td>1,004,614.48</td>
<td>7.47</td>
<td>1,089,011.30</td>
<td>7.18</td>
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<td>3</td>
<td>1,122,296.55</td>
<td>4.73</td>
<td>1,156,188.06</td>
<td>7.30</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>1,069,004.61</td>
<td>5.66</td>
<td>1,098,048.07</td>
<td>8.31</td>
</tr>
<tr>
<td>± 20</td>
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<td>11.55</td>
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</tbody>
</table>

**Figure 7.** Processing time and number of viable iterations.
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**Literature Cited**


