International Journal of Finance and Accounting 2019, 8(1): 23-35
DOI: 10.5923/j.ijfa.20190801.02

Intertemporal Analysis on the Technical Efficiency of Northeast Municipal Expenditure with Basic Education: A DEA Approach and Malmquist's Index

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Abstract The objective of this study is to measure and analyze the static and dynamic efficiency of Northeastern municipalities in educational expenditures for the years 2007 and 2013. In order to reach the objectives, a cluster analysis was first carried out using the non-hierarchical k-means method to the Northeastern municipalities according to socioeconomic and populational characteristics. After the groups were defined, the DEA-BCC model was applied to analyze the static efficiency and DEA-Malmquist to analyze the efficiency dynamics in the period. The results indicate that Northeastern municipalities improved efficiency in public spending on education in the period 2007 and 2013. However, it still maintains low levels of efficiency.

Keywords Education, Public spending, IDEB, DEA-Malmquist, Efficiency, Brazil

1. Introduction

From the beginning of the twentieth century to the present day, public spending has increased considerably in the major world economies. This growth, to a large extent, is associated with the welfare state policy, where governments provide basic public services (education, health, housing and protection) to the population.

Tanzi and Schuknecht (1997), Afonso and St Aubyn (2004) and Benicio et al. (2015), questioned the ability of governments to maintain their spending over a long period, especially in times of crisis, such as the current slowdown in the world economy, where governments are faced with declining tax revenues and growing social demand. This has required governments to make the best use of their resources, ie to make their spending efficient.

In Brazil, there is an increase in public spending, with attention being drawn to the increase in education spending in recent years. This increase in public spending on education is due to the fact that the Brazilian State, in the last two decades, has shown great interest in its educational development. For, with the promulgation of the 1988 Constitution, the universalization of basic education was established, becoming the right of everyone and the duty of the State, being the responsibility of the three federative entities (Union, States and Municipalities) to maintain the Brazilian education system in fully functioning.

The necessary resources destined to the maintenance of the Brazilian education system come from the tax revenues of the federative entities. It is the responsibility of the Union to allocate at least 18% of the Net Tax Revenue (RLT) and the States and Municipalities 25% of the RLT for maintenance and development of education (Benicio et al., 2015). In 2007, the Union created the Fund for the Maintenance and Development of Basic Education and Valorization of Education Professionals (FUNDEB), whose objective is to allocate resources for the financing of elementary education, early childhood education, secondary education and youth and adult education, which are offered by the states and municipalities (FNDE, 2012).

In 2008, the Union spent with FUNDEB about 0.54% of the RLT, representing an amount of 4.43 billion reais. In 2014, this figure rose considerably to 1.07% of RLT, an

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Published online at http://journal.sapub.org/ijfa

amount of 10.86 billion reais, which shows a considerable growth of 145.15% (Mendes, 2015). These figures point out the interest of the Brazilian State in strengthening basic education and, thus, developing the country's human capital, since, according to UNESCO (1998), basic education is the basis that gives the support for the formation of capital of a nation.

However, what has been verified is that the volume of resources employed in education alone does not guarantee the expected return. To verify the quality of basic education, the Basic Education Development Index (IDEB) is used to evaluate the performance of primary school students. Analyzing the most recent results (2013), it was verified that the Northeast presented the worst result for the initial years, note 4 (on a scale of 0 to 10), well below the South Region, which obtained a score of 5.6. The note for Brazil was 4.9 (INEP, 2013).

Given the amount of resources applied in basic education and the low results presented by Northeastern municipalities in the IDEB assessments, a study is needed to measure (and evaluate) the level of efficiency in municipal expenditure with education, as well as to analyze its behavior in a given period of time. And thus, present results that can signal the quality of public spending with education in the Northeastern municipalities. In the current literature, the most used method to measure efficiency in education is data envelopment analysis (DEA), the obtained results allow to guide the decision making units (DMUs) to optimize their results.

The great majority of national studies that deal with efficiency in public spending on education only make static analyzes, that is, do not evaluate the behavior of efficiency over time. In the national literature, one can highlight the works Rosano-Peña et al. (2012) and Santos et al. (2015), which analyzed the efficiency dynamics of municipal spending on education, the first for Goiás state and the second for Minas Gerais. No study was found that analyzed the dynamic efficiency of Northeastern municipalities with education expenditures. Thus, the present work can to the literature with application of dynamic efficiency analysis models.

To perform this work, the DEA-Malmquist model was used, which will allow to evaluate the efficiency of the Northeastern municipalities in spending on education between 2007 and 2013. The results will signal whether municipalities are making the application of these resources more efficient or less efficient.

This paper is structured in six sections, including this introduction. In the following section, the trajectory of public spending in the world and in Brazil will be presented, highlighting the expenses with education. In section three, the models used to measure technical efficiency with education expenditures are placed. Section four discusses the model that will be used in this work. In section five, the results found in the paper are presented and in the last section, final considerations of the work are made.

2. Public Expenditure

The role of the state in the economy has been thoroughly debated over the last three centuries. As Keynes (1926) points out, the incompetence and corruption of governments in the eighteenth century led many contemporary thinkers of this century and the next century to create lines of thought in which the state should act only with its minimal functions. Thus, leaving the economy on the market itself, without regulations, inaugurating the thought of laissez-faire.

Tanzi and Schuknecht (1997, p. 165) argue that nineteenth-century thinkers defended the minimal state, limiting the state to developing only allocative functions: "defense, law and order, basic public works, protection of property rights, and other similar functions". Until now, social issues were alien to the state. The authors confirm this situation when analyzing the public expenditure of several countries (Germany, United Kingdom, United States, Sweden, France, Japan), where public spending on average between 1870 and 1913 was found to be around 11% and 12% per year, in relation to the Gross Domestic Product (GDP).

In the twentieth century, the state began to worry not only about its allocative functions, but also with actions that minimize the distortions caused by the free market. In this sense, Musgrave (1959) defines the attributions of government in three functions: allocative, stabilizing and distributive. In the allocative function, the government acts in the sectors of goods and services that the market would not be able to offer in quantity satisfactory, or would not be economically viable, the so-called public goods. In the stabilizing function, government interferes in the economy by trying to prevent economic oscillations from affecting income and consumption, thereby reducing the welfare of families. In the distributive function, the government adopts measures to minimize the income distortions that the market generates.

After 1913, there was a change in the attributions of the State, providing the Social-State. Governments began to expand their obligations with: education, offering education at all levels; health; social security; and public assistance for unemployed individuals. Due to the politics of the Social State and the wars, there was a significant increase in public spending in relation to GDP during the 20th century. In Germany, the United Kingdom, the United States, France and Japan, public expenditures on average in 1913 went from 11.43% to 23.79% in 1937, reaching 44.83% in 1990 (Tanzi and Schuknecht, 1997). It is noteworthy that public spending, even in periods when the world was not at war, continued to increase.

The growth of public spending in the twentieth century can be analyzed according to the thinking of Adolf Wagner (1892), that as countries become more industrialized the demands for goods and services would grow. Thus, with the increase in per capita income in these countries, society's demand for goods and services would increase more than proportionately to income growth, putting pressure on public

spending. Wagner's Law is justified by three assumptions: i - the natural growth of government administrative activities and security spending; ii - industrialization and urbanization of the economy pressure governments to increase the supply of goods and services, such as education and health; iii - and as countries industrialize, the state should act to correct or mitigate market failures, such as monopolies and oligopolies (Read, 2015, Benicio et al., 2015).

Updating the data of Tanzi and Schuknecht (1997), in table 1 it can be observed that the public expenditures in 102 years almost quadrupled. This increase can be credited to a large extent by the new duties that governments have adopted to provide welfare state policy. Among the countries analyzed it is noted that, on average, there is a slight reduction in public spending between 2001 and 2007, from 40.7% to 40.4%. However, France, the United States and the United Kingdom continued to increase their public spending.

Still in table 1, there is a considerable increase in public spending in the period between 2007 and 2010 of 5.1 percentage points, an increase of 12.62%, a very considerable growth for a short period of time. This was due to the 2008 financial crisis, which reached the United States, where governments had to inject large amounts of public resources to stabilize markets.

Analyzing public expenditures by development groups, it is noted that in the first decade of this century, public spending followed a growth trend. There has been an abrupt increase in spending between 2008 and 2010 in the European Union, G7 and Developed Countries, corresponding to an increase of respectively 3.7, 2.7 and 2.5 percentage points. Over the same period, public expenditures in Brazil, Latin America, the Caribbean and the BRICS increased 1.4, 1.7 and 1.4 percentage points, respectively. This relationship can be seen in table 2.

Public spending in Brazil, in the last fifteen years, increased by 5.7 percentage points, that is, a total variation of 15.75% in the period. When compared to the growth of European Union spending (3.95%), G7 (6.87%) and Advanced Economies (5.54%), it is estimated that public spending in Brazil grew significantly more than the countries from these groups. It is worth mentioning that Brazil's public expenditures in 2001 corresponded to 36.2% of GDP, close to the average of the developed countries that make up the G7 (37.2%) and the Advanced Economies group (37.2%). In the year 2015, these countries reached an expenditure level of respectively 41.9%, 39.8% and 39.3%.

Comparing the pattern of public spending in Brazil with countries with economic similarities, it can be seen that public spending in Latin America and the Caribbean (34.9%) and BRICS (34.4%) are much lower than in Brazil (41.9%). It should be noted that Brazil is part of the BRICS, that is, the level of public spending in Brazil raises the average BRICS spending. As can be seen, the level of public spending in Brazil is in the pattern of developed countries.

2.1. Public Expenses with Education in Brazil

In the last two decades, the Brazilian government began to pay more attention to the problems of the country's low level of education. A major landmark was the promulgation of the 1988 Federal Constitution, which guaranteed the right to basic public education for all. It is worth noting that another considerable gain for Brazilian education was the National Education Guidelines and Bases Law (LDB, Law No. 9.394 / 1996), which established the guidelines that guide the Brazilian educational system (Araújo-Junior et al., 2016).

As observed in Table 2, Brazilian public spending at the beginning of this century followed a growth trend. In part, it can be attributed to increased spending on education. As Mendes (2015) points out, in 2004 the government spent 4% of the Treasury's net revenue on education, by the year 2014 this figure had risen to 9.3%. A significant growth of 130%.

Table 1. Evolution of total government expenditure in relation to GDP between 1913 and 2015(%)

Countries	1913	1920	1937	1960	1990	2001	2007	2010	2015
France	17	27,6	29	34,6	49,8	51,2	52,2	56,4	56,9
Germany	14,8	25	34,1	32,4	45,1	46,8	42,6	47,0	43,9
Japan	8,3	14,8	25,4	17,5	31,7	36,4	33,3	38,9	39,3
UK	12,7	26,2	30	32,2	39,9	36,5	39,4	45,4	40,2
EUA	7,5	12,1	19,7	27	33,3	32,7	34,5	40,0	35,7
Average	12,1	21,1	27,6	28,7	39,9	40,7	40,4	45,5	43,2

Source: International Monetary Fund (2016) and Tanzi and Schuknecht (1997).

Table 2. Evolution of total public expenditure by development groups and Brazil, in relation to GDP, between 2001 and 2015

Countries	2001	2006	2008	2010	2013	2014	2015
European Union	45.0	44.9	45.7	49.4	47.9	47.7	46.8
Brazil	36.2	39.2	37.4	38.8	37.5	39.1	41.9
G7	37.2	38.0	40.4	43.1	40.9	40.4	39.8
Developed Countries	37.2	37.7	39.9	42.5	40.4	40.0	39.3
Latin America and the Caribbean	26.7	29.2	30.5	32.2	32.7	33.7	34.9
BRICS	27.4	28.1	30.1	31.5	32.3	32.5	34.4

Source: International Monetary Fund (2016).

This growth in public spending on education can be attributed to the actions that governments have been taking to increase the level of education of the Brazilian population. For example, Constitutional Amendment No. 53/2006, which replaces the Fund for Maintenance and Development of Basic Education (Fundef) by the Fund for Maintenance and Development of Basic Education and Appreciation of Education Professionals (Fundeb). This amendment expanded the resources transferred from the Union to the states and municipalities for application in basic education (Benício; Rodopoulos; Bardella, 2015).

Brazil's public spending in 2012 with education corresponds to 5.4% of GDP, a high level, above the developed countries as: France (4.9%); South Korea (4.7%); United States (4.7%); and above the average pattern of public spending on education in OECD countries, 4.7%. This relationship can be observed in Figure 1.

Notably, Brazil has been giving significant attention to education. This situation can still be ratified when analyzing the share of public spending on education in total public expenditure. In 2012, in Brazil 17.2% of the total expenditure was for education, a figure well above that of countries such as: France (8.8%); Belgium (11%); Norway (14.1%); Korea (14.5%); and the average for the OECD countries (11.8%).

Even with this volume of resources destined for education, Brazil still leaves much to be desired in its results. In the International Student Assessment Program (PISA), developed by the OECD, Brazil does not present satisfactory results. In the last evaluation, Brazil occupied the last positions. Table 3 shows the PISA result by area. The worst performance in Brazil was in math with 392 points, 102 points lower than the OECD average. In terms of reading, it added 409 and in science, 405, falling below the OECD in 88 and 96 points, respectively.

Table 3. Result of PISA by area, for year 2012

Countires	Reading	Science	Mathematics
Korea	537	538	553
Bélgium	509	505	515
France	505	499	495
Norway	505	495	489
OCDE	497	501	494
EUA	498	497	481
Chile	441	445	475
Mexico	423	415	413
Brazil	409	405	392

Souce: OCDE (2016c, 2016d, 2016e).

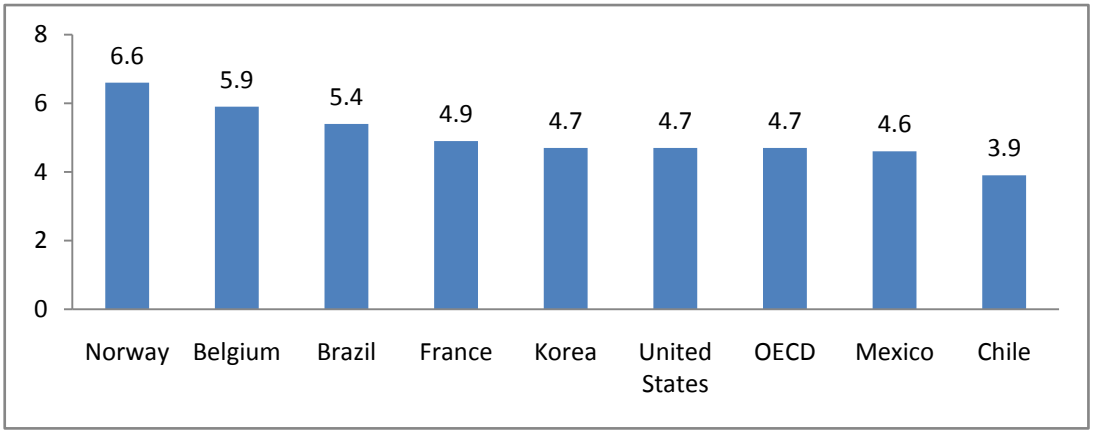

Figure 1. Public expenditure on education (%) in relation to GDP, for the year 2012 (*Source: OECD, (2016a)*)

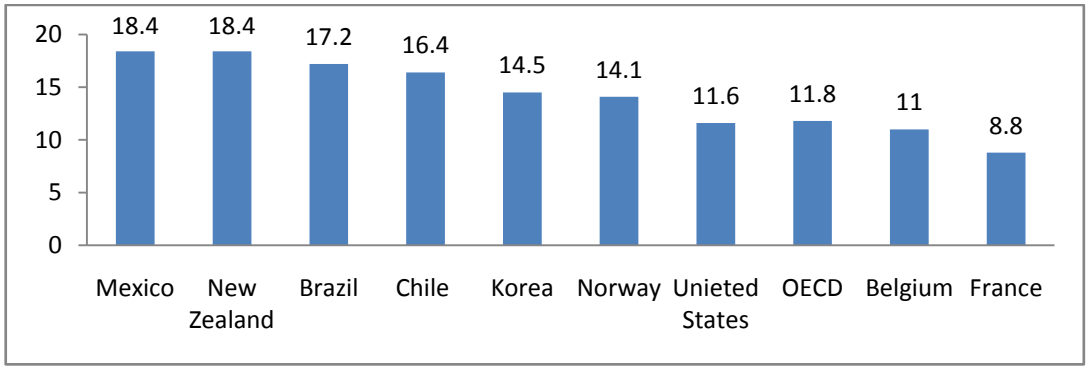

Figure 2. Public expenditure on education (%) in relation to total public expenditure, for the year 2012 (*Source: OECD, (2016b)*)

The interpretation of this Brazilian situation requires great care. Several factors may be responsible for the fact that Brazil presents high standards of public spending on education, while presenting poor results in the quality of education. Socioeconomic issues directly influence these results, Araújo Junior et al. (2016), show that the social vulnerability of the students negatively affects their performance. Another fact is the government's inefficiency in allocating resources, ensuring a maximization of results. Ribeiro (2008) points out that Brazil presents a high level of inefficiency in public spending when compared to some Latin American countries.

3. Methodology

In order to measure the level of efficiency of public spending, the use of non-parametric models has been recurrent. The works that deal with efficiency, mostly concentrate on analyzing spending on health and education. As Afonso and St. Aubyn (2004) argue, these two sectors have a large share of public spending.

In order to evaluate the efficiency of public spending, data envelopment analysis (DEA) and its extensions have been widely used. We can highlight the work of Afonso, Schuknecht and Tanzi (2005, 2010), who measured the efficiency of public spending in OECD countries; Afonso and St Aubyn (2004) who analyzed the efficiency of public spending on education and health in the OECD countries; Pang, et al. (2005) and Aristovnik (2012) who measured the efficiency of public spending on health and education in developing countries; Agasisti (2014) has already measured the dynamics of efficiency in public spending on education in the European Union; Prasetyo et al. (2013) evaluated the dynamics of efficiency in public expenditure on health and education in 81 countries; and Ribeiro (2008) measured the efficiency of public spending in Brazil and Latin America in health and education.

In the national literature, we can highlight: Faria et al. (2008) analyzed the efficiency of municipal expenditures on health and education in the state of Rio de Janeiro; Rosano-Pena et al. (2012) evaluated the dynamic efficiency of public spending on education in the municipalities of the State of Goiás, during the period 2005-2009; Gonçalves and Franca (2013) measured the efficiency of municipal expenditures with education in Brazil; Almeida and Gasparini (2011) evaluated the efficiency of municipal public spending in the State of Paraíba; and Rocha et al. (2015) analyzed the efficiency in the provision of education and health in Brazilian municipalities.

The following are extensions of the DEA model and its. Showing an overview of the model and its adaptations.

3.1. Intertemporal Analysis of Technical Efficiency Scores - DEA-BCC-Malmquist

The DEA method allows to calculate the technical efficiency for a period t. To make a dynamic analysis of the behavior of the DMUs in periods t and t+1, an intertemporal model is necessary. Rosano-Peña et al. (2012), emphasize the importance of using an intertemporal method in the DEA[1], since it allows observing the behavior of a certain DMU in other periods and the positioning of it in relation to the other DMUs.

There are several methods to calculate total factor productivity in more than one period, such as the use of the Laspeyres, Paasche, Fisher, Tornqvist and Malmquist. When using the DEA model, Malmquist is preferable since it does not require price information. Another advantage of the Malmquist Index is the possibility of decomposing it, providing information about changes in technical efficiency (pairing) and technological progress (frontier displacement) (Ferreira and Gomes, 2009).

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Thus, we chose to use the Malmquist Index to analyze the dynamics of technical efficiency in education expenditures in northeastern municipalities between periods t and t+1.

Malmquist Index.

The Malmquist Index was developed by Caves et al. (1982), based on the work of Sten Malmquist (1953). The index is calculated considering the quotient between the distance function in period t and t+1. There is the possibility of choosing which type of input / output to be given for distance functions.

As Färe et al. (1994) pointed out the distance function with output orientation is presented in function (1), where the period is overwritten and the orientation is subscript.

$$D_o^t(x^t, y^t) = (max\{\phi: (x^t, \theta y^t) \in S^t\})^{-1} \quad (1)$$

In function (1), ϕ is the minimum factor that the product can be contracted, remaining technically efficient, given the technology used, in period t; y^t t is the output, ie, output at period t; x^t t are inputs used in period t; S^t t is the set of production given the technology of period t. The set S^t t can be presented as follows:

$$S^t = \{(x^t, y^t): x^t \; can \; produce \; y^t \} \quad (2)$$

The function (1) represents the maximum expansion of the vector y^t (output) given the vector x^t (input). The distance function will be D_o^t (x^t, xy^t) = 1, if and only if (x^t, y^t) is at the production frontier. Thus, the technology applied in the combination (x^t, y^t) must be optimal for technical

1 In practice, according Ji and Lee (2010) the most of the available DEA programs use the dual forms. $\min_{\theta,\lambda} \theta$ subject to θxj − Xλ ≥ 0, Y λ ≥ yj , and λ ≥ 0, where λ is a semipositive vector in Rk and θ is a real variable. The computational procedure can be expressed as $\min_\theta \theta$. And the two-stage DEA model solves it. Efficiency Change (SEC) was used.

efficiency.

The distance function for more than one output-oriented period can be presented as follows:

$$D_o^t(x^{t+1}, y^{t+1}) = (max\{\phi: (x^{t+1}, \theta y^{t+1}) \in S^{t+1}\})^{-1} \quad (3)$$

The Malmquist productivity index is given by the ratio of the distance function of period t + 1 to the distance function in t, based on period t. It can also be based on the period t+1 (Färe et al., 1992).

The output-oriented Malmquist Index is expressed by:

$$M_o(x^{t+1}, y^{t+1}, x^t, y^t) = \left[\left(\frac{D_o^t(x^{t+1}, y^{t+1})}{D_o^t(x^t, y^t)}\right)\left(\frac{D_o^{t+1}(x^{t+1}, y^{t+1})}{D_o^{t+1}(x^t, y^t)}\right)\right]^{\frac{1}{2}} \quad (4)$$

Färe et al. (1992), show that equation (4) is equivalent to:

$$M_o(x^{t+1}, y^{t+1}, x^t, y^t) = \left(\frac{D_o^t(x^{t+1}, y^{t+1})}{D_o^t(x^t, y^t)}\right)\left[\left(\frac{D_o^t(x^{t+1}, y^{t+1})}{D_o^t(x^t, y^t)}\right)\left(\frac{D_o^{t+1}(x^{t+1}, y^{t+1})}{D_o^{t+1}(x^t, y^t)}\right)\right]^{\frac{1}{2}} \quad (5)$$

The result of equation (5) may be greater, equal or smaller than one. The interpretations of the results can be as follows:

- $M_o > 1$: Indicating that an increase in the productivity of DMU_k in period t + 1 in relation to t;
- $M_o = 1$: The productivity of DMU_k remained constant in period t + 1 in relation to t;
- $M_o < 1$: Indicating that the productivity of DMU_k decreased in period t + 1 in relation to t.

As already mentioned, the M_o index can be decomposed allowing the analysis of the dynamics of technical efficiency and efficient border behavior.

By decomposing equation (5), we can capture two effects: catch-up effect, which identifies whether the technical efficiency of the DMU improved, remained constant or worsened in the period t + 1 in relation to t; and the effect of the frontier-shift effect in period t + 1 with respect to t. This fact is due to the incorporation of new technologies (or reduction), allowing to analyze if there was technological progress (return).

Equation (6) shows the catch-up effect:

$$EE_o = \left(\frac{D_o^t(x^{t+1}, y^{t+1})}{D_o^t(x^t, y^t)}\right) \quad (6)$$

On what,

- $EE_o > 1$: Indicates that there was an increase in the technical efficiency of DMU_k in period t + 1 in relation to t;
- $EE_o = 1$:: The technical efficiency of DMU_k remained constant in period t + 1 in relation to t;
- $EE_o < 1$:: There was a reduction in the technical efficiency of DMU_k in period t + 1 in relation to t.

The frontier-shift effect is shown in equation (7):

$$ED_o = \left[\left(\frac{D_o^t(x^{t+1}, y^{t+1})}{D_o^t(x^t, y^t)}\right)\left(\frac{D_o^{t+1}(x^{t+1}, y^{t+1})}{D_o^{t+1}(x^t, y^t)}\right)\right]^{\frac{1}{2}} \quad (7)$$

On what,

- $ED_o > 1$:: Represents a technological progress of DMU_k in period t + 1 in relation to t;
- $ED_o = 1$: There were no technological advances of DMU_k in period t + 1 in relation to t;
- $ED_o < 1$: There was a technological regression of DMU_k in period t + 1 in relation to t.

DEA BCC- Malmquist

Färe et al. (1992) proposes a nonparametric model based on the inputs / outputs that makes it possible to calculate the total factor productivity (TFP) dynamics, combining the Malmquist Index proposed by Caves et al. (1982) and the idea of efficiency measure developed by Farrell (1957), being the same idea of efficiency used in the DEA models.

Färe et al. (1994) developed a method to calculate Malmquist productivity using DEA. In his work, we considered the output-oriented model with constant returns to scale. However, it allows the possibility of calculating the Malmquist productivity with variable returns of scale, including in the model the condition of convexity.

Assuming that for each DMU_k (k = 1, ..., n) an output vector is produced $y_k^t = (y_{1k}^t, \dots, y_{sk}^t$ using an input vector $x_k^t = (x_{1k}^t, \dots, x_{mk}^t)$ for each time period T, t = 1, ..., T. The Malmquist productivity index (and its decompositions) will be calculated using the DEA-BCC model with output orientation.

The productivity index becomes:

$$M_o = \left(\frac{\phi_0^t(x_0^t, y_0^t)}{\phi_0^{t+1}(x_0^{t+1}, y_0^{t+1})}\right)\left[\frac{\phi_0^{t+1}(x_0^{t+1}, y_0^{t+1})}{\phi_0^t(x_0^{t+1}, y_0^{t+1})}\frac{\phi_0^{t+1}(x_0^t, y_0^t)}{\phi_0^t(x_0^t, y_0^t)}\right]^{\frac{1}{2}} \quad (8)$$

The effect of pairing and the displacement of the boundary will be given by (9) and (10), respectively:

$$EE_o = \left(\frac{\phi_0^t(x_0^t, y_0^t)}{\phi_0^{t+1}(x_0^{t+1}, y_0^{t+1})}\right) \quad (9)$$

$$ED_o = \left[\frac{\phi_0^{t+1}(x_0^{t+1}, y_0^{t+1})}{\phi_0^t(x_0^{t+1}, y_0^{t+1})}\frac{\phi_0^{t+1}(x_0^t, y_0^t)^{\frac{1}{2}}}{\phi_0^t(x_0^t, y_0^t)}\right] \quad (10)$$

3.2. Cluster Analysis Using Non-hierarchical k-means Models Formed from the Euclidean Distance

In order to perform the cluster analysis, it is first necessary to create an indicator of similarity between the DMUs, which in this case will be the Euclidean distance.

We obtain a similarity indicator for DMU that will be used to form K groups, with the non-hierarchical method k-means. According to Mingoti (2005) the k-means method is composed of four steps: a) first define the k centroids to initialize the participation process; b) each element of the data set is compared with each initial centroid, given by expression (13), and thus each element is grouped by reference to the shortest distance; c) apply step b to each of the n sample elements, recalculate the centroid values for each new formed group, and repeat step b, considering the centroids of these new groups; and d) steps b and c must be repeated until all sample elements are well allocated in their groups.

3.3. Data

Choice of variables for Cluster Analysis

Cluster analysis aims to group similar samples within a universe, from common characteristics among the elements, forming homogeneous groups. Before estimating the DEA-BCC-Malmquist model, a grouping of northeastern municipalities was carried out considering similar characteristics of the municipalities.

The variables chosen to form the clusters with the municipalities should be related to the objective of the study, since the model requires that the DMUs possess the characteristics that are as similar as possible (De-White; López-Torres, 2015). In this case, the variables used are:

- Municipal Human Development Index (IDHM) - the HDI is an indicator of quality of life and economic development that uses three indicators: health, education and income. With this variable, it is intended to group the municipalities according to their socioeconomic characteristics;
- PIB per capita - with this variable we intend to group the municipalities according to their degree of economic activity;
- Population - with this variable we intend to group the municipalities considering the size of their population;

FIRJAN Municipal Development Index (IFDM) - is an index that analyzes the development of the municipality considering three areas of action: income and employment; health and education. With this variable we intend to group the municipalities according to their socioeconomic characteristics.

Choice of inputs and outputs

According to Mello et al. (2005), the selection of inputs and outputs should be careful not to err in the DEA estimation. The variables chosen should be linked to the research objective.

To estimate the DEA-VRS-Malmquist model, the municipal expenditure with basic education per student was used as input. To construct this variable, we considered the ratio between the total expenditure of the municipality with elementary education and the number of students enrolled in this same level of basic education.

As an output, IDEB's grades were considered for the beginning and end years. The IDEB grade is calculated by INEP, by combining the results of the Brazil Test (Portuguese and Mathematics) with the student approval rate.

In order to carry out this work, the information from the 1794 Northeastern municipalities available on the official websites was used. Chart 1 shows the variables used for the formation of clusters and for the estimation of the Malmquist Index.

Initially all databases were compiled on only one basis, totaling 1794 observations. Subsequently, the observations that lacked information were withdrawn, closing the database with 1105 observations. We used Stata 15.1 program.

Chart 1. Description of the variables used to form the clusters and estimation of the Malmquist index

Variable	Description	Source
Variables used to define clusters		
pib_per_cap	Gross domestic product of the municipality divided by the number of inhabitants. In (R$)	National Accounts 2010, IBGE
População	Number of residents in the municipality	Demographic Census 2010
IDHM	Municipal Human Development Index	PNUD 2010
IFDM	FIRJAN Municipal Development Index	FIRJAN 2010
Variables used in the DEA-Malmquist model		
ideb_final_2007	Index of Basic Education Development of the final years in the year 2007	Index of Basic Education Development of the final years in the year 2007
ideb_final_2013	Index of Basic Education Development of the final years in the year 2013	IDEB Microdata 2013
ideb_inicial_2007	Index of Basic Education Development of the initial years in the year 2007	IDEB Microdata 2007
ideb_inicial_2013	Basic Education Development Index of the initial years in the year 2013	IDEB Microdata 2013
gastos_educ_aluno_2007	Municipal expenditure with basic education in 2007. In (R$)	IDEB Microdata 2007 and FINBRA 2007
gastos_educ_aluno_2013	Municipal spending on basic education in 2013. In (R$)	IDEB Microdata 2007 and FINBRA 2007

Source: Author's own elaboration.

4. Results

In this section we will discuss the results achieved in this work. In order to analyze the static and temporal efficiency of municipal expenditures with education, the Northeastern municipalities were first divided into homogeneous groups, using the non-hierarchical k-means method. The DEA-BCC model was then estimated to measure the technical efficiency level for the years 2007 and 2013. The Malmquist-DEA method was used to analyze the efficiency behavior from 2007 through 2013.

4.1. Formation of Clusters

To estimate the DEA-Malmquist model, the Northeastern municipalities were first divided into homogeneous groups. For this, the cluster analysis method was applied. For the formation of the groups, variables were considered that grouped the municipalities by socioeconomic aspects, size and level of development.

The non-hierarchical k-means method requires a prior definition of the group quantity. Mufti et al. (2005) and Halpin (2016), suggest the Calinski and Harabasz test to determine the optimal number of groups, since this indicator analyzes similarity and dissimilarity, within and between groups.

The Calinski-Harabasz test analyzes the centroid of each group, evaluating the similarity within these groups. At the same time, it analyzes the distance of the centroids between the groups, calculating the dissimilarity between them and determining the ideal quantity of clusters.

To perform the Calinski and Harabasz test, the non-hierarchical k-means method was first applied six times. For each estimation different amounts of groups were defined, 2, 3, 4, 5, 6 and 7. Soon after, the Calinski and Harabasz test was performed for each estimation. By the test criteria, higher values indicate that the groups are better, that is, homogeneous within the group and heterogeneous between the groups.

The result of the Calinski and Harabasz test (Table 4), indicates that the use of five groups is the most appropriate. For, the Pseudo-F presented the highest value (585.43) for the group with five clusters.

Table 4. Calinski and Harabasz test result

Number of Clusters	*Pseudo-F*
2	472,37
3	494,28
4	422,59
5	585,43
6	543,59
7	512,41

Source: Author's own calculations.

From Table 5, Group 4 is composed of seven observations, being the following municipalities: São Luís, Fortaleza, Salvador, Natal, Teresina, João Pessoa and Recife. This group is constituted by seven capitals of the Northeast that have characteristics distinct from the other cities. On average, the population of this group is 1,431,695 inhabitants, well above the average of the other groups. It is of great importance that these municipalities are grouped together, since it is expected that the large urban centers will have more modern systems of control and application of resources.

Groups 1 and 3 have characteristics that represent most of the Northeastern municipalities. They are municipalities with small population, low GDP per capita and with minor socioeconomic indicators. Group 3 has the worst indicators, GDP per capita (R$ 4,525.86) is lower than the average of the municipalities in the Northeast (R$ 6,386.02), showing that these municipalities have a low economic activity in relation to the other municipalities of the region. The Municipal Human Development Index (IDHM) and the Municipal Development Index (IFDM) also presented below-average results, respectively, of 0.56 and 0.45, indicating that these municipalities have serious socioeconomic problems.

Table 5. Descriptive statistics of Clusters

Group	Freq.	Part.	Average			
			GDP per capita	Population	IDHM	IFDM
1	503	46%	5.310,73	19.008	0,60	0,56
2	135	12%	10.441,66	85.200	0,66	0,64
3	433	39%	4.525,86	16.453	0,56	0,45
4	7	1%	16.503,26	1.431.695	0,76	0,75
5	27	2%	35.658,69	48.082	0,65	0,62
NE	1105	100%	6.386,02	35.753	0,52	0,59

Source: Author's own elaboration, based on IBGE (2010), FIRJAN (2010) and UNDP (2010) data.

Group 5 has very different characteristics from the other groups, with a GDP per capita (R$ 35,658.69) well above the Northeast and the indicators of HDI (0.65) and IFDM (0.62) are above average. This Group is made up of municipalities that have a high economic activity, for example: Camaçari -BA, has an Industrial Pole that counts on petrochemical, chemical and automobile companies; Ipojuca-PE and Cabo de Santo Agostinho - PE, which houses the Suape Industrial and Port Complex.

Thus, it can be concluded that: Group 1 is formed by small municipalities, with low economic activity; Group 2 is made up of small and medium-sized municipalities with relevant economic activity; Group 3 is composed of small municipalities, with low economic activity and low socioeconomic indicators; Group 4 is formed by large municipalities with high economic activity and high socioeconomic indicators; and Group 5, is composed of small and medium-sized municipalities with high economic activity and relevant socioeconomic indicators.

As can be seen, groups can be considered homogeneous. With this, the DEA-Malmquist model can be applied without incurring the problem of non-homogeneity pointed out by

Dyson et al. (2001).

Static efficiency of municipal spending on education in the years 2007 and 2013

In order to measure the technical efficiency level of Northeastern municipalities with spending on education for the years 2007 and 2013, the DEA-BCC model with product orientation was applied. Three variables, one input and two outputs, were used. As an output variable, the municipal IDEB scores were considered for the initial and final years of elementary school. And as input (input) the municipal expenditure with education per student.

Table 6. Variables used in the DEA-BCC and Malmquist-DEA model, IDEB result for the initial and final years and municipal expenditure on education per student

Groups	Average					
	Early years		Final Years		Expenditure per student (R$)	
	2007	2013	2007	2013	2007	2013
Group 1	3.21	4.25	2.91	3.44	1938.74	4555.96
Group 2	3.42	4.35	2.99	3.54	1776.43	3916.86
Group 3	3.03	3.83	2.75	3.17	1638.83	4033.35
Group 4	3.80	4.39	3.03	3.56	1836.88	3389.93
Group 5	3.20	3.98	2.80	3.11	2408.94	4882.44
Northeast	3.33	4.16	2.90	3.36	1919.97	4155.71

Source: Author's own elaboration, based on information from INEP (2007; 2013) and FINBRA (2007; 2013).

On average, Northeastern municipalities (Table 6) spent about R$ 1,919.97 per student with education in 2007, in 2013 this figure was R$ 4,882.44, an increase of 116%. Group 5 presented spending on education well above average for both periods. This value can be justified by the fact that the municipalities that make up this group have a high economic activity. And the Federal Constitution of 1988 obliges municipalities to invest at least 25% of income from taxes on education. Thus, these municipalities have more resources to invest in education.

Still on education spending, it is observed that in Group 4, even though it is made up of cities with high economic activity, the amounts spent on education per student are lower than the average in the two periods, R$ 1,836.88 in 2007 and R$ 3,389.93 in 2013.

It can be seen that the IDEB scores improved significantly from 2007 to 2013. The average IDEB of the Northeastern municipalities in the year 2007 for the initial years was 3.33, to 4.16 in 2013, approximately a growth of 25%. In the final years, the IDEB of 2007 was 2.9 and in 2013, 3.36, a growth of 15.86%. Group 4 presented results above the northeastern average in both periods, in which, in the year 2007 for the initial years was of 3.8, and for the final years of 3.03. In 2013, respectively, this score was 4.39 and 3.56.

As can be seen, the result of the municipal IDEB between the period of 2007 and 2013 has improved significantly. However, the level of technical efficiency of municipalities with spending on education was very low. Analyzing the DEA-BCC result (Table 7) together a high level of inefficiency is observed for the two periods.

In the year of 2007, about 98.19% of the municipalities in the Northeast were considered inefficient, and in 2013 they were 97.83%. It can still be noted that approximately 80% of the municipalities (in both periods) had efficiency levels below 0.8. In 2007, only 1.81% of municipalities were considered efficient, in 2013 this number was 2.17%.

These high levels of inefficiency of the Northeastern municipalities with expenditures on education are related to poor management of public resources. As Gasparini and Miranda (2011) points out, the municipalities in the Northeast have an average level of efficiency in public spending of 50%. This low level of efficiency generated a waste of public resources of approximately R$ 3.7 billion in the year 2000.

Table 7. Distribution of technical efficiency with orientation to the output of Northeastern municipalities for the years 2007 and 2013

Groups	Period	Efficiency strata (%)				
		0|-------0,6	0,6|-----0,8	0,8-----|0,9	0,9|------1	1
NE	2007	14.76	64.95	14.31	4.17	1.81
	2013	23.64	56.61	12.59	4.98	2.17
Group 1	2007	15.11	68.59	12.72	2.58	0.99
	2013	31.61	50.30	12.33	4.57	1.19
Group 2	2007	1.49	60.45	27.61	7.46	2.99
	2013	23.13	53.73	12.69	5.97	4.48
Group 3	2007	19.63	64.20	11.32	3.46	1.39
	2013	16.17	65.82	11.78	4.85	1.39
Group 4	2007	-	14.29	14.29	42.86	28.57
	2013	-	-	42.86	28.57	28.57
Group 5	2007	-	44.44	25.93	18.52	11.11
	2013	3.70	55.56	22.22	3.70	14.81

Source: Author's own elaboration, based on the results of the research.

Analyzing by groups, a similar pattern is observed between Groups 1, 2 and 3 and the Northeast. Over 95% of municipalities were inefficient for both periods. Group 1 had the worst results; more than 80% had efficiency levels below 0.8 in both periods. In 2007, less than 1% of municipalities were considered efficient and in 2013 they were 1.19%. Although it is tempting, it cannot be said that there is an improvement in the Group 1 efficiency indicators, because according to Santos et al. (2015), it is not correct to consider that there was an improvement in the period of 2013 compared to 2007, since they are operating at different borders.

Group 4 presented the best results in both periods, in which 28.57% were considered efficient. The cities of Recife-PE and Salvador-BA achieved the best results in this group, operating at full efficiency in both periods. In 2007 the city of Natal-RN presented the worst level of efficiency in the group, being 77.7%. For Natal to become efficient, it would have to increase the IDEB score by 29.88%, keeping education expenses per student unchanged.

In 2007, about 11% of the municipalities that make up Group 5 were considered efficient, in 2013 they were 14.81%. The Bahia cities of Conceição do Jacuípe and Camaçari, achieved full efficiency in both periods, at the same time that more than 70% of the municipalities operated on an efficiency scale of less than 70%. In 2007, the city Porto do Mangue-RN presented the worst performance with efficiency level (64%) and in 2013 it was Itagibá-BA (59%). For these municipalities to reach the efficiency frontier, it will be necessary to increase the grade of the IDEB, respectively, by 56% and 69%, keeping constant the expenses with education.

In order to explain the variation in efficiency levels in municipal expenditures with education between groups, we can highlight the work of Rocha et al. (2015), which analyzed the efficiency of the Brazilian municipalities with education expenditures, grouping the municipalities according to the size of the population. Coming to the conclusion that small municipalities (up to 50 thousand inhabitants) operate with the average level of efficiency in municipal expenses with education of 50.3% and large municipalities (more than 500 thousand inhabitants) operate with 81.2%.

We can see a pattern between the results found in this study and Rocha et al. (2015). Groups 1, 2 and 3 are small cities and have the worst results. One justification for this fact is that the smaller municipalities have per capita costs well above the big cities. As can be seen in Table 6, per capita education expenditures for Groups 1, 2 and 3 are higher than those of G 4. These results reinforce the need for a cluster analysis.

Northeastern municipalities presented low levels of efficiency in spending on education. When analyzed in a disaggregated form, all groups present unsatisfactory results. It is also noticed that there is an increase in the number of municipalities that operate in the efficiency frontier, in relation to 2007 to 2013. We cannot affirm that there was an improvement in efficiency levels, this information will be analyzed in the next section with the results of the model Malmquist-DEA.

Intertemporal analysis of the technical efficiency of Northeastern municipalities with spending on education between 2007 and 2013

In this section the results of the Malmquist-DEA model will be presented. The Malmquist index analyzes the productivity of a given municipality over time, stating whether in the evaluated period the municipality won, lost or maintained the level of productivity. The Malmquist index can be decomposed into two indexes: the catch-up effect that measures the behavior of technical efficiency over time, indicating whether the efficiency in municipal public spending with education has improved, worsened or remained constant; and the effect of the frontier-shift effect, which makes it possible to evaluate whether there was a technological progress or regression in the period.

Table 8 shows the distribution of municipalities according to the results of the Malmquist index and their decomposition. Considering all municipalities in the Northeast, 93.7% of municipalities increased productivity with education expenditures between 2007 and 2013. It can be considered that the improvement in productivity was due to the increase in efficiency of municipalities, 99% gained efficiency in the period. The productivity of the municipalities could have presented better results; however, there was a technological retrogression in the period.

Table 8. Distribution of the behavior of the municipalities according to the results of the Malmquist index, pairing effect and the effect of the border shift in the period between 2007 and 2013

Groups	Municipalities Distribution (%)						
	Productivity		Technical Efficiency			Technological Change	
	Won	Lost	Won	Kept	Lost	Incorporated	Stepped back
Northeast	93.7	6.3	99.0	0.01	0.99	-	100.00
Group 1	97.6	2.4	99.9	-	0.01	-	100.00
Group 2	94.8	5.2	63.29	16.46	20.25	100.00	-
Group 3	97.5	2.5	43.88	-	56.12	100.00	-
Group 4	100.0	-	42.86	28.57	28.57	100.00	-
Group 5	92.6	7.4	66.67	7.41	25.93	100.00	-

Source: Author's own calculations.

Analyzing in a disaggregated way, all groups increased productivity considerably in public spending on education, from 2007 through 2013. In Group 1, 97.6% of the municipalities obtained gains in productivity. This increase was due to the fact that 99% of municipalities improved efficiency in education spending. It is observed that there was a technological regression in the municipalities, that is, the municipalities failed to incorporate new technologies that would improve public spending on education.

More than 90% of the municipalities that comprise Groups 2, 3, 4 and 5 managed to increase productivity in education spending between 2007 and 2013. The improvement in productivity was mainly due to the technological gains obtained by all the municipalities that make up these groups. Since less than 67% of these municipalities managed to improve efficiency in the period.

In Table 9, the results of the Malmquist index, the pairing effect and the effect of the border shift are presented. Analyzing Northeastern municipalities in an aggregate way, it has been observed that, on average, they increased efficiency with education expenditures by 244% in the period. However, productivity increased by an average of only 31%, due to the technological retrogression, on average of 62%, in the period.

Table 9. Result of the Malmquist index, pairing effect and the effect of the border shift in the period between 2007 and 2013

Groups	Average		
	Index of change of productivity	Efficiency change	Technological Change
Northeast	1.31	3.44	0.38
Group 1	1.39	2.80	0.50
Group 2	1.83	0.78	2.35
Group 3	2.01	1.64	1.22
Group 4	1.59	1.09	1.45
Group 5	1.68	1.07	1.56

Source: Author's own calculations.

The municipalities that make up Group 1, increased on average the efficiency of education spending by 180%. Productivity grew by only 39%, due to a technological backwardness of 50%. Group 3 presented (on average) the best gain in productivity (101%), this was due to the improvement in efficiency (64%) of the municipalities and the technological progress (22%) presented in the period.

The municipalities of Group 4, obtained the best technological progress in the period (135%). Indicating that the municipal managers were able to introduce new technological processes that allowed the optimization in the application of public resources in education.

The productivity of municipal expenditures with education on average improved considerably in the period evaluated, 2007/2013. A justification for this improvement is presented by Rosano-Peña et al. (2012), stressing that productivity gains in education spending may be related to the availability of municipal external evaluation results (IDEB, Prova Brasil), pressing municipal managers to incorporate new technologies in order to achieve the best results.

As can be seen, municipalities have significantly improved the efficiency of public spending on education in the period 2007-2013. However, municipalities maintain low levels of efficiency. To illustrate, the municipality of São José do Brejo da Cruz-PB (Group 1) improved the efficiency of public spending on education in the period by 160%. However, in the year 2013 it operated with the efficiency level of 55%. This situation reflects in the great majority of municipalities analyzed, requiring municipal public managers more commitment in the application of public resources.

5. Conclusions

Meeting the growing social demand for goods and services on a limited budget has been a major challenge faced by several countries in the world. With Brazil no different, the economic crisis experienced in the last three years has exposed the difficulties of governments in providing basic services to the population.

In the last 15 years, Brazilian public spending has increased considerably. During this period there is a real increase in spending on education, indicating that governments have paid more attention to education. However, indicators of teaching quality have presented timid results. This situation refers to the need to measure and analyze the level of efficiency in public expenditure on education.

Basic education is considered one of the main variables responsible for the development of a country. In Brazil, municipalities are responsible for providing basic education, through the transfer of the Union and the States. With this, it is of extreme importance that we evaluate how resources for basic education are being applied.

The objective of this study was to evaluate the static and dynamic efficiency of public education expenditures in the Northeastern municipalities for the years 2007 and 2013. And how they behaved during this period. It is worth mentioning that the choice of Northeastern municipalities was mainly due to the low results presented in the Basic Education Development Index (IDEB).

Analyzing the Northeastern municipalities in an aggregated and disaggregated way (by groups) it is observed that all presented low levels of efficiency. Groups 1, 2 and 3 obtained the worst results, more than 90% of the municipalities that make up these groups were considered inefficient. These groups are small municipalities with low socioeconomic indicators. On the other hand, Group 4 presented the best results, this group is constituted by large cities and with high socioeconomic indicators.

It can be assumed that municipalities have certain characteristics influence municipal efficiency in the application of public resources. Larger municipalities were

able to introduce new technologies (processes or methods) much more than small municipalities. These new technologies help municipalities optimize results. In this case, technology should be understood as any action taken by municipal managers that improve (or maintain) educational indicators by maintaining (or reducing) the inputs used in the process.

The productivity and efficiency of Northeastern municipalities with education spending improved during the period 2007-2013. However, there may be a technological regression in municipalities in the application of public resources in education. Indicating, that municipalities are using much more resources (compared to 2007) and reaching relatively lower result.

The great majority of Northeastern municipalities managed to considerably increase technical efficiency in the period. However, when comparing the efficiency gain and the level of efficiency that the municipalities are operating, one realizes that the situation of most municipalities is not good. The gain in technical efficiency obtained in the period, still does not allow municipalities to position themselves on the efficiency frontier. However, the municipalities are indicating a sense of approximation of the efficiency frontier.

It is concluded that, the work reached its objectives in measuring the level of technical efficiency in the municipal expenses with education, between 2007 and 2013. It is suggested for future research that the following questions are answered: what are the factors that influence the improvement or worse efficiency of municipal spending on education.

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