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ESTIMATION OF THE WEIBULL DISTRIBUTION *FORM* PARAMETER AS A FUNCTION OF THE *SCALE* PARAMETER BY THE PERCENTILE METHOD

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Resumo

Estimativa do parâmetro forma *da distribuição Weibull em função do parâmetro* escala *pelo método dos percentis*. Dentre as funções densidade de probabilidade para estimar e projetar distribuições diamétricas, a função Weibull é uma das mais experimentadas, sendo o método dos percentis uma alternativa para estimar seus parâmetros, com diferentes deduções para cálculo dos parâmetros de locação, escala e forma, algumas exigindo iteração entre as fórmulas. Neste trabalho, utilizou-se o método dos percentis e estabeleceu – se uma função entre o parâmetro de forma, os percentis assintóticos e o parâmetro de escala. Esta função foi testada com dados de Inventário Florestal Contínuo (IFC) em um povoamento de *Khaya grandifoliola*, e em renques de *Eucalyptus* spp. em sistema ILPF e, também foi validada em distribuições projetadas pela aplicação de uma metodologia de recuperação dos parâmetros para projeção de crescimento e produção de florestas (C&PF). Os resultados mostraram uma correlação exata, quando comparada com os parâmetros calculados pelo método dos percentis. This function using the percentile method, due to its practicality, may be an alternative to obtain the shape parameter in C&PF methodologies.
Palavras-chave: manejo florestal, fdp, F(x), prognose.

Abstract

Among the probability density functions that estimate and project diametric distributions, the Weibull function is one of the most studied, and the percentile method is an alternative approach for estimating its parameters, with different deduction methods for calculating the location, scale, and shape parameters; in addition, of these parameters, some require iteration between formulas. In this study, the percentile method was used, and a function was established between the shape parameter, the asymptotic percentiles, and the scale parameter. This function was tested with continuous forest inventory (CFI) data of a *Khaya grandifoliola* stand and *Eucalyptus* spp. in integrated forest-livestock-forest (IFLF) systems, and the function was also validated based on distributions designed by the application of a recovery methodology for determining forest growth and production (FG&P) parameters. The results showed an exact correlation when compared with the parameters calculated by the percentile method. This function using the percentile method, due to its practicality, may be an alternative for obtaining the shape parameter in FG&P methodologies.
Keywords: forest management, pdf, F (x), prognosis.

INTRODUCTION

The study of probabilistic functions in the forest environment mainly aims to model the diameter distribution and the projection of forest growth through forest growth and production (FG&P) models (OLIVEIRA *et al.*, 2009; MIGUEL *et al.*, 2010; BINOTI *et al.*, 2012; RETSLAFF *et al.*, 2012; LEITE *et al.*, 2013; POUDEL; CAO, 2013; AZEVEDO *et al.*, 2016; JESUS *et al.*, 2017; MIRANDA *et al.*, 2018; OGANA; CHUKWU; AJAYI, 2020). These models are essential to forest management because they allow the determination of forest dimensions at a future age for stock planning and economic analyses.

Among the probability density functions (pdfs), which aim to estimate and project the frequency of trees by size class, the Weibull function is among the most studied, and it performs better when compared to the performance of other pdfs due to its ability to flexibly and easily estimate functions and parameters (BAILEY; DELL, 1973; LIMA; BATISTA; PRADO, 2014; RIBEIRO *et al.*, 2014; MADI *et al.*, 2017).

The importance of the Weibull function in terms of modeling stochastic problems from numerous application areas is described in Murthy, Xie and Jiang (2003), who classified their modifications and organized them into a taxonomy of the Weibull models. Through this taxonomy, it is possible to identify variations, applications, and methods that can be used to obtain its parameters. The best-known methods for parameter estimation are percentiles, maximum likelihood, moments, and regression (BERGER; LAWRENCE, 1974; CAO, 2004; HUDAK; TIRYAKIOGLU, 2009), with percentiles generally being the simplest parameter used that does not result in a loss of efficiency; however, these different deductions methods calculate location, scale, and shape parameters, and some require iterations between the formulas (BAILEY; DELL, 1973; CAO, 2004; MIGUEL *et al.*, 2010; WENDLING; EMERENCIANO; HOSOKAWA, 2011; ORELLANA *et al.*, 2017). Modern techniques, such as the metaheuristic genetic algorithm and simulated annealing, were compared to the percentile and moment methods and performed better (ARAÚJO *et al.*, 2021), but these newer techniques are still minimally explored in the literature compared to the classical methods.

Most applications with probabilistic functions in FG&P models use data from the continuous forest inventories (CFIs) through plots measured over time with the following purpose: to model the structure of the observed data, usually the diameter at breast height (DBH); to obtain attributes of plots by age, relating estimated parameters of the pdf with forest attributes; and to project at a future age the pdf parameters as a function of the forest attributes also projected. This whole process is performed by empirical functions between attributes and parameters, using the parameter prediction method (WANG *et al.*, 2011) or the parameter recovery method with moments or percentiles (ARCE, 2004; RETSLAFF *et al.*, 2012; POUDEL; CAO, 2013).

The purpose of this study is to validate a function that can be used in FG&P models that integrates the shape parameter, the asymptotic percentiles, and the scale parameter, using the percentile method, observed data from CFIs, and data projected by the parameter recovery method.

MATERIALS AND METHODS

Development

The Weibull probability density function with three parameters is the following:

$$f(x) = \frac{c}{b}\left(\frac{x-a}{b}\right)^{c-1} e^{-\left(\frac{x-a}{b}\right)^c},$$

where a is the location parameter, b is the scale parameter, c is the shape parameter, and x is the random variable being studied. To integrate *f(x)* and obtain its cumulative function, *F(x)*, for simplicity, we consider the Weibull function of two parameters (scale and shape) as follows:

$$F(x) = \int fxdx = \int \frac{c}{b}\left(\frac{x}{b}\right)^{c-1} e^{-\left(\frac{x}{b}\right)^c} dx \qquad (1)$$

Applying the rule: $\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$, and $\left(\frac{u}{v}\right)' = \frac{u'.v-u.v'}{v^2}$ and, doing $u = -\left(\frac{x}{b}\right)^c = -\frac{x^c}{b^c}$, we have

$$\frac{du}{dx} = -\frac{cx^{c-1}.b^c}{(b^c)^2} = -\frac{cx^{c-1}}{b^{c-1}.b} = -\frac{c}{b}\left(\frac{x}{b}\right)^{c-1} \text{ or } du = -\frac{c}{b}\left(\frac{x}{b}\right)^{c-1} dx.$$

Substituting du into Eq.1, we obtain the cumulative function, *F(x)*:

$$\mathrm{F(x)} = \int -\mathrm{e^u du} = 1\text{-}\mathrm{e^u} + \mathrm{c} = 1\text{-}\mathrm{e}^{-\left(\frac{x}{b}\right)^c} \qquad (2)$$

or, for a cumulative function with 3 parameters: $\mathrm{P} = \mathrm{F(x)} = 1\text{-}\mathrm{e}^{-\left(\frac{x-a}{b}\right)^c}$

The formula for calculating the percentiles is deduced from Eq.2, which is the cumulative probability up to the determined percentile, as follows.

$$P = 1 - e^{-\left(\frac{x}{b}\right)^c} \rightarrow 1 - P = e^{-\left(\frac{x}{b}\right)^c} \rightarrow -\ln(1-P) = \left(\frac{x}{b}\right)^c \rightarrow$$

$$[-\ln(1-P)]^{\frac{1}{c}} = \frac{x}{b} \rightarrow [-\ln(1-P)]^{\frac{1}{c}} = \frac{x}{b} \rightarrow x = b[-\ln(1-P)]^{\frac{1}{c}}$$

For the Weibull 3P function, the diameter location parameter is subtracted from the percentile (Eq.3).

$$x - a = b[-\ln(1-P)]^{\frac{1}{c}} \qquad (3)$$

To calculate parameter a, choosing not to perform the iteration, the method in Wendling, Emerenciano and Hosokawa (2011), in which a is selected as a proportion of the minimum diameter (0.1, 0.2..., 1) by the Kolmogorov–Smirnov test, applies the differences between the observed and estimated cumulative distributions. To calculate the parameters b and c, two asymptotic percentiles, P_1 and P_2, respectively, are assigned to Eq.3, which now has two expressions for algebraic manipulation.

$$x_1 - a = b[-\ln(1-P_1)]^{\frac{1}{c}} \quad \text{and} \quad x_2 - a = b[-\ln(1-P_2)]^{\frac{1}{c}}$$

Parameter c is obtained by the ratio between the two percentiles, according to the following development:

$$\frac{x_1-a}{x_2-a} = \left[\frac{-\ln(1-P_1)}{-\ln(1-P_2)}\right]^{\frac{1}{c}} \qquad \ln\left(\frac{x_1-a}{x_2-a}\right) = \frac{1}{c} ln\left[\frac{-\ln(1-P_1)}{-\ln(1-P_2)}\right] \qquad c = \frac{ln\left[\frac{\ln(1-P_1)}{\ln(1-P_2)}\right]}{\ln\left(\frac{x_1-a}{x_2-a}\right)} \qquad (4)$$

Parameter b can be obtained by the difference between the percentiles as follows:

$$(x_2 - a) - (x_1 - a) = b[-\ln(1-P_2)]^{\frac{1}{c}} - b[-\ln(1-P_1)]^{\frac{1}{c}}$$

$$x_2 - x_1 = b\{[-\ln(1-P_2)]^{\frac{1}{c}} - [-\ln(1-P_1)]^{\frac{1}{c}}\}$$

$$b = \frac{x_2 - x_1}{[-\ln(1-P_2)]^{\frac{1}{c}} - [-\ln(1-P_1)]^{\frac{1}{c}}} \qquad (5)$$

Considering the ratio from Eq.5 and replacing Eq.4 without parameter *a* (Weibull 2P), this relationship becomes linear, according to Eq. 6, because it will depend on the increase in diameters x_1 and x_2 with age, referring to the asymptotic percentiles P_1 and P_2.

$$\frac{b}{x_2-x_1} = \frac{1}{[-\ln(1-P_1)]^{\left\{\frac{1}{\frac{\ln\left[\frac{\ln(1-P_1)}{\ln(1-P_2)}\right]}{\ln\left(\frac{x_1}{x_2}\right)}}\right\}} - [-\ln(1-P_2)]^{\left\{\frac{1}{\frac{ln\left[\frac{\ln(1-P_1)}{\ln(1-P_2)}\right]}{\ln\left(\frac{x_1}{x_2}\right)}}\right\}}} \qquad (6)$$

Thus, if *c* in Eq.4 and *b* and Eq.6 are functions of x_1, x_2, P_1, and P_2, the shape parameter (*c*) can be regressed as a function of the scale parameter (*b*) according to Eq.7.

$$c = \mathrm{b}_1 * \frac{b}{(x_2-x_1)} + b_0 \qquad (7)$$

Data

The validation of Eq.7 was performed in two forest systems, an African mahogany (*Khaya grandifoliola*) stand and eucalyptus hedges (*Eucalyptus urophylla x E. grandis clone*) in integrated forest-livestock-forest (IFLF) systems, both implemented in the municipality of Sete Lagoas, Minas Gerais, Brazil.

The plantations followed the fertilization recommendations for potential production. The mahogany seedlings were drip irrigated in the first two years because the climate of the Sete Lagoas region is seasonal, the Cwa type, with a dry season in the winter (May to October). The average annual rainfall is 1,335 mm, approximately 70% of which occurs between October and February (PEEL; FINLAYSON; MCMAHON, 2007). Eucalyptus plantations in this region do not require irrigation, but when planting in 2009, it was necessary to irrigate in the 1st year because planting occurred late, in early February.

The mahogany was planted on 12/01/2009 with 5 x 5 m spacing. A portion of 50 trees was selected from the CFI. The diameter was measured at 1.3 m height (DBH in cm) and total height (h in m) in the respective months reported in Table 1.

The eucalyptus was planted with 15 x 2 m spacing in two IFLF systems on two dates: the first on 02/05/2009 and the second on 10/24/2011. Both systems were subjected to 50% systematic thinning in half of the area in 2015. In each row, one tree was selected from every 10 trees, sampling 40 trees per system, with the diameter measured at 1.3 m in height (DBH in cm) and total height (m). The measurement periods for both systems are shown in Table 1.

Table 1. Date and month of the CFI of the forest systems.
Tabela 1. Datas e meses de mensuração do inventário florestal contínuo para os sistemas florestais.

Mahogany		Euc2009		Euc2011	
Date	Months	Date	Months	Date	Months
01/05/2012	29.0	04/06/2012	39.9	06/12/2013	25.8
01/05/2013	41.0	24/04/2013	50.6	13/11/2014	37.2
01/05/2014	53.0	27/05/2014	63.7	26/11/2015	49.8
17/05/2015	65.5	09/07/2015	77.1	03/10/2016	60.2
12/06/2016	78.4	03/10/2016	92.0	21/11/2017	74.0
04/06/2017	89.7	14/08/2017	102.3	27/11/2018	86.4
05/10/2018	106.2	29/05/2018	111.8	12/11/2019	98.0

To perform the projections, the CFI data were divided into seven combinations (six for three dates and one with all dates), without including the projection date (Table 2).

Table 2. Date combinations of the CFI of the forest systems.
Tabela 2. Combinações de datas de mensuração do inventário florestal contínuo para os sistemas florestais.

Combinations	
Mahogany and Euc 2009	Euc2011
2012, 2013, 2014	2013, 2014, 2015
2013, 2014, 2015	2014, 2015, 2016
2014, 2015, 2016	2015, 2016, 2017
2015, 2016, 2017	2016, 2017, 2018
2012, 2014, 2016	2013, 2015, 2017
2013, 2015, 2017	2014, 2016, 2018
2012-2017	2013-2018

The mahogany and eucalyptus planted in 2009 were projected for 2018, the eucalyptus planted in 2011 was projected for 2019, and the thinned eucalyptus plantations planted in 2009 and 2011 were projected for 2019.

The calculations of the CFI attributes, the parameters of the observed and estimated distributions by age, and the projection model applied were performed using an Excel Visual Basic Application (VBA) algorithm. The 24th and 93rd percentiles (P_1 and P_2) were used to estimate the location, scale, and shape parameters of the Weibull function by the percentile method using the calculations in Wendling, Emerenciano and Hosokawa (2011) for parameter a and Eq. 4 and Eq. 5 for parameters c and b from the observed distributions, with a diameter class amplitude of 1 cm. The recovery of parameters a, b, and c for the projected distributions was performed by the functions in Eq. 8, using the age of the trees in months (I) and the diameters in the asymptotic percentiles (x_1 and x_2).

$$(x_1, x_2) = \mathrm{b}_0 + \mathrm{b}_1 * ln(\mathrm{I}) \qquad (8)$$
$$a = \mathrm{b}_0 + \mathrm{b}_1 * x_1$$
$$b = \mathrm{b}_0 + \mathrm{b}_1 * (x_2 - a)$$
$$c = \mathrm{b}_1 * \frac{b}{(x_2 - x_1)} + b_0$$

With the values of b and c and the respective diameters in the asymptotic percentiles generated per year for each forest system (mahogany, eucalyptus 2009, 2011 not thinned and thinned, original, and projected), the regression equations were fitted by Eq.7.

RESULTS

Table 3 shows the asymptotic diameters x_1 and x_2 (in percentiles 24 and 93) determined from the observed frequency distributions and the values of a, b, and c calculated by the percentile method. It is expected that the values of a and b will increase with age. The location parameter defines the beginning of the distribution, which will increase each year due to diameter growth, and parameter b will increase with age due to the increase in the dispersion of the diameters caused by the distance between diameters of suppressed trees, codominant trees, and dominant trees. Table 3 shows this trend with some exceptions. Conversely, parameter c has no presumed behavior with age or correlation with the other parameters and percentiles individually.

Table 3. Diameters x_1 and x_2 in percentiles 24 and 93, and parameters a, b, and c estimated by percentile method.
Tabela 3. Diâmetros x_1 e x_2 nos percentis 24 e 93, parâmetros a, b e c estimados pelo método dos percentis.

	Year	x_1	x_2	a	b	c
Mahogany						
2009	2012	3.80	5.70	0.00	4.79	5.60
	2013	6.70	9.50	0.00	8.17	6.50
	2014	8.90	12.70	0.00	10.90	6.39
	2015	11.80	15.00	0.00	13.53	9.46
	2016	13.10	17.80	0.00	15.60	7.41
	2017	15.00	19.70	0.00	17.52	8.33
	2018	16.20	22.00	0.00	19.28	7.42
Euc 2009	2012	17.05	19.85	15.55	2.73	2.16
	2013	18.80	21.90	17.60	2.48	1.78
	2014	19.90	23.40	17.70	3.78	2.39
	2015	22.05	26.85	19.00	5.22	2.40
	2016	22.00	26.10	16.24	7.82	4.22
	2017	23.00	27.20	10.70	14.54	7.73
	2018	24.90	29.80	20.07	7.20	3.24
Euc 2011	2013	12.95	16.65	10.53	4.10	2.45
	2014	16.75	20.50	11.79	6.83	4.03
	2015	16.60	20.55	13.15	5.33	2.98
	2016	17.10	20.70	13.70	5.13	3.14
	2017	17.85	21.70	14.65	5.02	2.88
	2018	20.05	25.15	16.15	6.28	2.72

	2019	20.65	25.15	16.70	6.09	2.99
Euc 2009 thinning	2016	24.45	30.30	21.20	5.84	2.21
	2017	25.00	31.75	21.80	6.10	2.00
	2018	28.70	34.65	24.90	6.50	2.41
	2019	29.20	37.00	25.20	7.41	2.10
Euc 2011 thinning	2016	19.10	23.30	15.25	5.86	3.08
	2017	20.75	25.55	12.15	11.07	5.12
	2018	23.75	31.50	19.45	7.73	2.20
	2019	25.00	30.20	20.60	6.90	3.00

For the growth projection, where regression functions are included for attributes x_1 and x_2, a, b, and c, according to Eq. 8, which were fitted with data from the years of each combination (IFC years), are provided with the coefficients of determination of each equation generated (Table 4). The results of R^2 for c were all equal to 1. Exact adjustments for a reasonable number of repetitions are not expected in empirical functions, which demonstrates the possibility that Eq.7 is a deterministic function.

Table 4. Coefficients of determination obtained for the parameters recovered for the projection results in Table 3.
Tabela 4. Coeficientes de determinação obtidos na recuperação dos parâmetros para os resultados de projeção da Tabela 3.

	Years of IFC	x_1	x_2	a	b	c
Mahogany	2012-13-14	1.000	0.999	0.994	1.000	1.000
	13-14-15	0.982	0.998	0.994	0.993	1.000
	14-15-16	0.972	0.989	0.954	0.984	1.000
	15-16-17	0.968	0.999	0.988	0.992	1.000
	12-14-16	0.996	0.999	0.997	1.000	1.000
	13-15-17	1.000	0.996	0.983	0.998	1.000
	12-13-14-15-16-17	0.993	0.997	0.987	0.998	1.000
Euc 2009	2012-13-14	0.985	0.993	0.881	0.967	1.000
	13-14-15	0.945	0.925	0.926	0.992	1.000
	14-15-16	**0.753**	**0.577**	**0.000**	0.967	1.000
	15-16-17	**0.576**	**0.031**	0.865	0.998	1.000
	12-14-16	1.000	1.000	**0.157**	0.998	1.000
	13-15-17	0.963	0.881	**0.303**	0.995	1.000
	12-13-14-15-16-17	0.969	0.931	**0.097**	0.994	1.000
Euc 2011	2013-14-15	0.978	0.995	0.998	0.995	1.000
	14-15-16	0.918	0.870	0.997	0.975	1.000
	15-16-17	0.992	0.863	0.999	0.787	1.000
	16-17-18	0.876	0.855	0.978	0.990	1.000
	13-15-17	0.951	0.935	0.990	1.000	1.000
	14-16-18	1.000	0.988	0.999	0.965	1.000
	13-14-15-16-17-18	0.97	0.956	0.993	0.986	1.000

Table 5 shows the values of the asymptotic diameters x_1 and x_2 projected and the parameters a, b, and c recovered at the projection age from the combination of dates in each forest system (CFI years). The accuracy of the a, b, and c values between the combinations was visually observed to be lower for the results of the 2009 Eucalyptus than for the other results.

Table 5. Diameters x_1 and x_2 in percentiles 24 and 93, and parameters *a*, *b*, and *c* projected for the combinations of the continuum forestry inventory (CFI) dates.

Tabela 5. Diâmetros x_1 e x_2 nos percentis 24 e 93, e parâmetros *a*, *b* e *c* projetados para as combinações de datas do Inventário Florestal Contínuo.

	CFI Years	x_1	x_2	*a*	*b*	*c*
Mahogany projected for 2018	2012-13-14	14.8	20.7	0.0	17.9	6.80
	13-14-15	16.8	20.7	0.0	18.9	10.80
	14-15-16	16.6	21.6	0.0	19.2	8.70
	15-16-17	16.4	22.2	0.0	19.5	7.60
	12-14-16	15.7	21.3	0.0	18.7	7.40
	13-15-17	16.8	21.6	0.0	19.4	9.00
	12-13-14-15-16-17	16.3	21.4	0.0	19.0	8.30
Euc 2009 projected for 2018	2012-13-14	23.7	28.1	21.0	4.9	2.50
	13-14-15	24.9	31.1	20.2	7.6	2.70
	14-15-16	23.7	28.6	17.6	8.6	4.00
	15-16-17	23.1	26.9	10.0	15.0	8.90
	12-14-16	23.4	27.9	17.2	8.6	4.30
	13-15-17	24.1	28.9	12.7	14.1	6.50
	12-13-14-15-16-17	23.9	28.6	14.8	11.7	5.50
Euc 2011 projected for 2019	2013-14-15	21.8	27.0	16.6	7.7	3.20
	14-15-16	20.6	24.3	16.2	6.1	3.60
	15-16-17	18.2	22.0	15.1	5.0	2.90
	16-17-18	19.5	24.3	15.8	6.0	2.76
	13-15-17	19.5	23.7	15.6	5.9	3.10
	14-16-18	19.9	24.6	16.0	6.1	2.80
	13-14-15-16-17-18	19.7	24.2	15.8	6.0	2.90
Euc 2009 thinning projected for 2019	2015-16-17	29.2	37	25.2	7.4	2.10
	16-17-18	30.1	36.5	26.1	6.5	2.30
Euc 2011 thinning projected for 2019	2015-16-17	23.9	29.2	12.7	14	5.80
	16-17-18	25	33.4	19.7	10.4	2.70

With the data from Tables 3 and 5, the ratio of *b* is obtained by the difference between x_2 and x_1 to obtain the regression equations presented in Table 6, which informs the coefficients of Eq. 7 for the forest system. The P value showed a b_0 significance lower than 5% for three results, and these results were close to zero. For the others, which were not significant at 10%, the line passes through the origin, with b_0 being eliminated from the equation. On the other hand, the slope is significant at the extreme, with its lowest significance at two thousandths.

The coefficients of determination, R^2, differ from 1 in the projected data because these data are being generated from attributes also projected from the CFI, implying less precision in the adjustment process.

Thus, it is inferred that b_0 is equal to zero and that 2.2 is an approximate constant for the value of b_1, with the expectation of being a value very close to the theoretical value of the relationship between *c* and *b*. The value of this constant should change with the choice of percentiles, which in this study were 24 and 93 because P_1 and P_2 are present in Eq.6.

Table 6. Regression of *c* as a function of $b/(x_1 - x_2)$ by forest system and P values for the coefficients b_0 and b_1 of the regression equations.

Tabela 6. Regressão de c em função de $b/(x_1 - x_2)$ por sistema florestal e valores de significância P para os coeficientes b_0 e b_1 das equações de regressão.

	b_0	P value	b_1	P value	R^2
Mahogany	-0.0996	1E-07	2.2623	4E-17	1
Euc 2009	-0.0334	0.026	2.2400	3E-12	1
Euc 2011	-0.0136	0.1014	2.2186	1E-12	1
Euc 2009 thinned	0.0334	0.0009	2.1763	3E-09	1
Euc 2011 thinned	-0.0264	0.3302	2.2309	0.0026	1
Projected mahogany	0.1031	0.6075	2.2101	1E-07	0.9975
Euc 2009 projected	-0.0154	0.8205	2.2437	5E-09	0.9993
Euc 2011 projected	0.0639	0.6443	2.1388	3E-06	0.9906
Euc 2009 thinned Pro + Euc2011 thinned Pro	0.0445	0.4697	2.1770	0.0002	0.9996
Mean	0.0060		2.2110		

Table 7 shows the results of *c* (*c_regres*) obtained by the regression equations in Table 6 and the values of *c* obtained by the constant b_1 = 2.2 (*c_aprox*), with very similar values by pairs. These data are plotted in Figure 1 to verify the possible trend by the use of a constant for *c* in Eq.7.

Table 7. Parameter *c_regres* estimated by linear regression as a function of the ratio between the parameter b and the difference in the diameters in the 24th and 93rd percentiles, estimated by the percentile method and projected to date combinations in the continuous forestry inventory (CFI) for the mahogany forest and the eucalyptus IFLF systems, without and with thinning, and the parameter *c_aprox* empirically obtained, considering b_0 =0 and b_1 = 2.2.

Tabela 7. Parâmetro *c_regres* estimado por regressão linear em função da razão entre o parâmetro *b* e a diferença dos diâmetros nos percentis 24 e 93, estimados pelo método dos percentis e projetados para as combinações de datas do Inventário Florestal Contínuo (IFC), para povoamentos do Mogno em monocultura e do Eucalipto em ILPF, sem e com desbate, e o parâmetro *c_aprox* obtido empiricamente, considerando b_0 =0 e b_1 = 2.2.

	Year	***c_regres***	***c_prox***		**CFI Years**	***c_regres***	***c_prox***
Mahogany 2009	2012	5.60	5.54	Mahogany	2012-13-14	6.81	6.67
	2013	6.50	6.42	projected	13-14-15	10.81	10.66
	2014	6.39	6.31	for 2018	14-15-16	8.59	8.45
	2015	9.46	9.30		15-16-17	7.53	7.40
	2016	7.41	7.30		12-14-16	7.48	7.35
	2017	8.33	8.20		13-15-17	9.04	8.89
	2018	7.42	7.31		12-13-14-15-16-17	8.34	8.20
Euc 2009	2012	2.15	2.15	Euc 2009	2012-13-14	2.48	2.45
	2013	1.76	1.76	projected	13-14-15	2.73	2.70
	2014	2.39	2.38	for 2018	14-15-16	3.92	3.86
	2015	2.40	2.39		15-16-17	8.84	8.68
	2016	4.24	4.20		12-14-16	4.27	4.20
	2017	7.72	7.62		13-15-17	6.58	6.46
	2018	3.26	3.23		12-13-14-15-16-17	5.57	5.48
Euc 2011	2013	2.45	2.44	Euc 2011	13-14-15	3.23	3.26
	2014	4.03	4.01	projected	14-15-16	3.59	3.63
	2015	2.98	2.97	for 2019	15-16-17	2.88	2.89
	2016	3.15	3.13		16-17-18	2.74	2.75
	2017	2.88	2.87		13-15-17	3.07	3.09
	2018	2.72	2.71		14-16-18	2.84	2.86
	2019	2.99	2.98		13-14-15-16-17-18	2.92	2.93
Euc 2009 thinning	2016	2.21	2.20	Euc 2009 thinning	15-16-17	2.11	2.09
	2017	2.00	1.99	projected for 2019	16-17-18	2.26	2.23
	2018	2.41	2.40				
	2019	2.10	2.09				
Euc 2011 thinning	2016	3.09	3.07	Euc 2011 thinning	15-16-17	5.80	5.81
	2017	5.12	5.07	projected for 2019	16-17-18	2.74	2.72
	2018	2.20	2.19				
	2019	2.93	2.92				

When plotting *c_regres* and *c_aprox* as a function of *c* obtained by the percentile method with the data in Table 7, a small bias is observed, informed by the positive relative error, with higher values of c_aprox up to approximately the value of c = 5 obtained by the percentile method; however, these values decrease when the error becomes negative (Figure 1). Notably, the slope may be slightly higher than b_1 = 2.2, which is not yet the exact constant of Eq. 7 (considering the coefficient b_0 = 0). The distances between the *c* estimates are more noticeable for higher values (Figure 1).

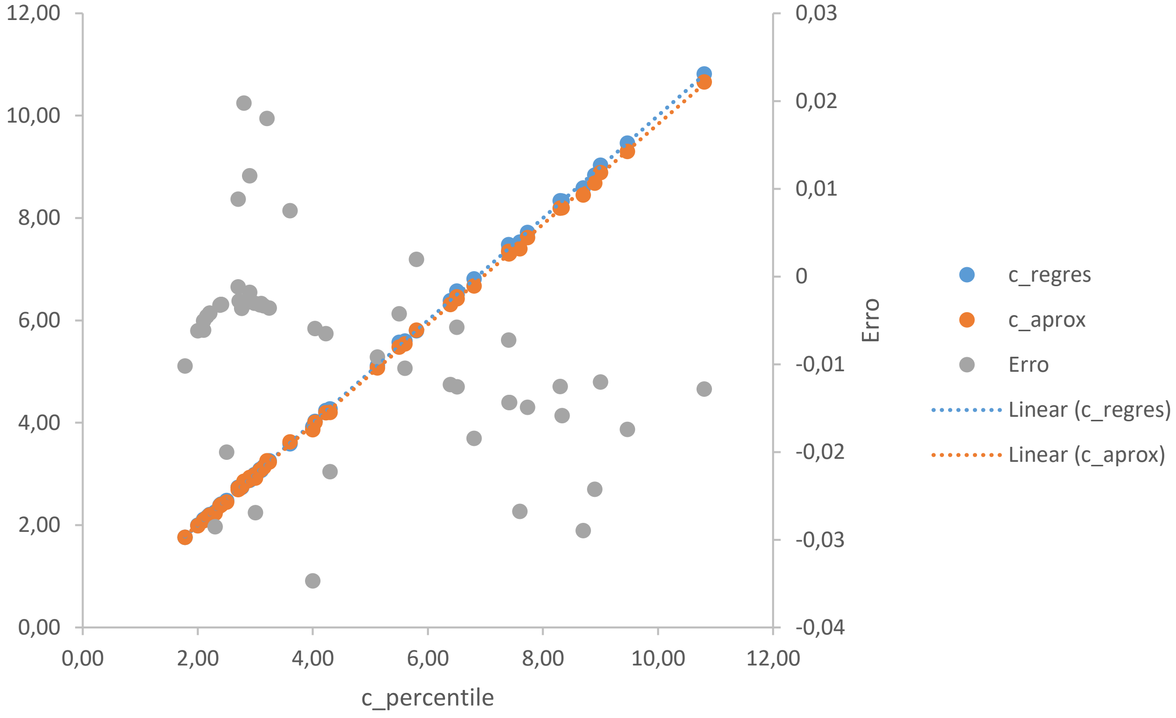

Figure 1. Plot of *c* obtained by the percentile method with *c_regress* obtained by linear regression with $b/(x_1 - x_2)$, *c_aprox* calculated by determining 2.2*[$b/(x_1 - x_2)$], and *error* = *(c_aprox – c)/c*

Figura 1. plot de *c* obtido pelo método dos percentis com *c_regres* obtido por regressão linear com $b/(x_1 - x_2)$, *c_aprox* calculado pela aproximação 2.2*[$b/(x_1 - x_2)$], e *erro* = *(c_aprox – c)/c*

DISCUSSION

The parameters x_1, x_2, *a*, *b*, and *c* of the Weibull function conceptually help in the functional interpretation of each function. For example, *a* and x_1 should show proportionality, varying with age, because the location parameter, where the function starts, will be on the left and proportionally close to x_1. Then, *a* can be linearly regressed with x_1 (Eq.8).

The parameter *b* informs the amplitude of the distribution so that it presents proportionality with the amplitude between *a* and x_2, which can be regressed by these two parameters (Eq.8). Finally, parameter *c*, which defines the shape of the distribution, shows an exact relationship with the ratio $b/(x_2\text{-}x_1)$, which indicates proportionality between amplitudes (Eq.8).

The limitation of the percentile method is the choice of asymptotic percentiles, according to the type of distribution. Not every distribution will have two asymptotic percentiles, as in the case of “exponential” distributions. It is also important to know which percentiles to use, and this will depend on the observed distribution. There are different values proposed for asymptotic percentiles in the literature when the distribution approaches a normal distribution (CAO, 2004; MIGUEL *et al.*, 2010; WENDLING; EMERENCIANO; HOSOKAWA, 2011; POUDEL; CAO, 2013).

On the other hand, capturing the value of the variables referring to the percentiles in the list of data is simple and easy to implement when the nature of the observed probabilistic distribution is adequate for obtaining both percentiles. For example, in native forests, diameter distributions follow an inverted J-type function, as in Lima, Batista and Prado (2014) and Orellana *et al.* (2017). In these cases, it is not possible to extract the 1st asymptotic percentile, nor is it possible to determine whether the function of Eq. 7 will have functionality, considering x_1 = 0. This analysis can be performed with data from native forests.

However, for planted forests, it is usually possible to obtain both percentiles, and this method becomes simple for estimating the parameters of the Weibull distribution (2P, 3P) with Eq. 4 and Eq. 5 and the Wendling method (WENDLING; EMERENCIANO; HOSOKAWA, 2011). To use this methodology in FG&P projection models, parameter recovery requires regression functions to estimate diameters for the asymptotic percentiles (CAO 2004; MIGUEL *et al.*, 2010; POUDEL; CAO, 2013). In this study, these functions were simplified according to the functions in Eq.8. If the function in Eq.7 remains efficient in other surveys, then the application of the set of functions in Eq. 8 for the FG&P model may provide another alternative, with greater simplicity.

CONCLUSIONS

This results of this study made it possible to conclude the following:

- In the function test between c, calculated by the percentile method, and $b/(x_2-x_1)$, the correlation is exact. As a deterministic function with a linear coefficient equal to zero, the slope, although not deduced in this study, is a constant close to 2.2.
- Projecting forest growth using this function can simplify the method for parameter recovery.

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REFERENCES

ARAÚJO, L. A.; OLIVEIRA, R. M.; DOBNER JR., M.; SILVA, C. S. J.; GOMIDE, L. R. Appropriate search techniques to estimate Weibull function parameters in a Pinus spp. plantation. **Journal of Forestry Research**, Harbin, v. 32, p. 2423-2435, 2021. DOI: https://doi.org/10.1007/s11676-020-01246-z.

ARCE, J. E. Modelagem da estrutura de florestas clonais de *Populus deltoides* marsh. Através de distribuições diamétricas probabilísticas. **Ciência Florestal**, Santa Maria, v. 14, n. 1, p. 149-164 149, 2004. DOI: https://doi.org/10.5902/198050981790.

AZEVEDO, G. B.; OLIVEIRA, E. K. B.; AZEVEDO, G. T. O. S.; BUCHMANN, E. P. M.; REZENDE, A. V. Modelagem da produção em nível de povoamento e por distribuição diamétrica em plantios de eucalipto. **Scientia Florestalis**, Piracicaba, v. 44, n. 110, p. 383-392, 2016. DOI: http://dx.doi.org/10.18671/scifor.v44n110.11.

BAILEY, R. L; DELL, T. R. Quantifying diameter distributions with the Weibull function. **Forest Science**, Bethesda, v. 19, n. 2, p. 97-104, 1973. DOI: https://doi.org/10.1093/fostscience/19.2.97.

BERGER, R. W.; LAWRENCE, K. Estimating Weibull parameters by linear and nonlinear regression. **Technometrics**, Oxford, v. 16, n. 4, p. 617-619, Nov. 1974.

BINOTI, D. H. B.; BINOTI, M. L. M. S.; LEITE, H. G.; MELIDO, R. C. N.; SANTOS, F. L. Descrição da distribuição diamétrica de sistemas agrossilvipastoris utilizando as funções Weibull e Hiperbólica. **Revista Árvore**, Viçosa, MG, v. 36, n. 2, p. 349-356, 2012. DOI: https://doi.org/10.1590/S0100-67622012000200016.

CAO, Q. V. Predicting parameters of a Weibull function for modeling diameter distribution. **Forest Science**, Bethesda, v. 50, n. 5, p. 682-685, 2004. DOI: https://doi.org/10.1093/forestscience/50.5.682.

HUDAK, D.; TIRYAKIOĞLU M. On estimating percentiles of the weibull distribution by the linear regression method. **Journal of Materials Science**, London, v. 44, n. 8, p. 1959-1964, 2009. DOI: http://dx.doi.org/10.1007/s10853-009-3306-1.

JESUS, C. M.; MIGUEL, E. P.; REZENDE, A. V.; GASPAR, R. O.; GATTO, A.; VALADÃO, M. B. X.; CARRIJO, J. V. N. Funções de densidade de probabilidade para estimativa das distribuições de variáveis dendrométricas em um povoamento clonal de eucalipto. **Espacios**, Caracas, v. 38, n. 16, p. 30, 2017. Disponível em: https://www.revistaespacios.com/a17v38n16/a17v38n16p30.pdf. Acesso em: 22 ago. 2022.

LEITE, H. G.; ALCÂNTARA, A. E. M.; BINOT, D. H. B.; OLIVEIRA NETO, R. O.; SILVA, M. L. M. Comparação entre predição e projeção da distribuição de diâmetros de povoamentos de eucalipto submetidos a desbastes. **Revista Árvore**, Viçosa, MG, v. 37, n. 2, p. 321-328, 2013. DOI: https://doi.org/10.1590/S0100-67622013000200013.

LIMA, R. A. F.; BATISTA, J. L. F.; PRADO, P. I. Modeling tree diameter distributions in natural forests: an evaluation of 10 statistical models. **Forest Science**, Bethesda, v. 61, n. 2, p. 320-327, 2014. DOI: https://doi.org/10.5849/forsci.14-070.

MADI, J. P. S.; VENDRUSCOLO, D. G. S.; SILVA, C. A.; CARVALHO, M. P. L. C.; CARVALHO, S. P. C. Univariate models to represent the diametric distribution of thinned stand of *Tectona grandis* Linn. F. **Advances in Forestry Science**, Cuiabá, v. 4, n. 2, p. 119-123, 2017. DOI: https://doi.org/10.34062/afs.v4i2.4726.

MIGUEL, E. P.; MACHADO, S. A.; FIGUEIREDO FILHO, A.; ARCE, J. E. Using the Weibull function for prognosis of yield by diameter class in *Eucaliptus urophylla* stands. **Cerne**, Lavras, v. 16, n. 1, p. 94-104, 2010. DOI: https://doi.org/10.1590/S0104-77602010000100011.

MURTHY, D. N. P.; XIE, M.; JIANG, R. **Weibull models**. New York: Wiley, 2003. 416 p.

MIRANDA, R.; FIORENTIN, L.; PÉLLICO NETTO, S.; JUVANHOL, R.; CORTE, A. D. Prediction system for diameter distribution and wood production of eucalyptus. **Floresta e Ambiente**, Seropédica, v. 25, n. 3, p. 12, 2018. DOI: http://dx.doi.org/10.1590/2179-8087.054816.

OGANA, F. N.; CHUKWU, O.; AJAYI, S. Tree size distribution modelling: moving from complexity to finite mixture. **Journal of Forest and Environmental Science**, Gangwon-do, v. 36, n. 1, p. 7-16, 2020. DOI: https://doi.org/10.7747/JFES.2020.36.1.7.

OLIVEIRA, M. L. R.; LEITE, H. G.; NOGUEIRA, G. S.; CAMPOS, J. C. C. Modelagem e prognose em povoamentos não desbastados de clones de eucalipto. **Revista Árvore**, Viçosa, MG, v. 33, n. 5, p. 841-852, 2009. DOI: http://dx.doi.org/10.1590/S0100-67622009000500007.

ORELLANA, E.; FIGUEIREDO FILHO, A.; PÉLLICO NETO, S.; DIAS, A. N. Métodos de ajuste e procedimentos de seleção de funções probabilísticas para modelar a distribuição diamétrica em floresta nativa de araucária. **Ciência Florestal**, Santa Maria, v. 27, n. 3, p. 969-980, 2017. DOI: http://dx.doi.org/10.5902/1980509828668.

PEEL, M. C.; FINLAYSON, B. L.; MCMAHON, T. A. Updated world map of the Koppen-Geiger climate classification. **Hydrology and Earth System Sciences**, v. 11, n. 5, p. 1633-1644, 2007. DOI: https://doi.org/10.5194/hess-11-1633-2007.

POUDEL, K. P.; CAO, Q. V. Evaluation of methods to predict Weibull parameters for characterizing diameter distributions. **Forest Science**, Bethesda, v. 59, n. 2, p. 243-252, 2013.

RETSLAFF, F. A. S.; FIGUEIREDO FILHO, A.; DIAS, A. N.; BERNETT, L. G.; FIGURA, M. A. Prognose do crescimento e da produção em classes de diâmetro para povoamentos desbastados de *Eucalyptus grandis* no sul do Brasil. **Revista Árvore**, Viçosa, MG, v. 36, n. 4, p. 719-732, 2012. DOI: https://doi.org/10.1590/S0100-67622012000400013.

RIBEIRO, A.; FERRAZ FILHO, A. C.; SCOLFORO, J. R. S.; PÉLLICO NETO, S.; MACHADO, S. A. Estrutura da distribuição diamétrica em plantio experimental de candeia (*Eremanthus erythropappus* (dc.) Macleish). **Ciência Florestal**, Santa Maria, v. 24, n. 4, p. 1055-1065, 2014. DOI: https://doi.org/10.1590/1980-509820142404024.

WANG, W.; PENG, C.; ZHANG, S. Y.; ZHOU, X.; LAROCQUE, G. R.; KNEESHAW, D. D.; LEI, X. D. Development of TRIPLEX-Management model for simulating the response of forest growth to pre-commercial thinning. **Ecological Modelling**, Amsterdam, v. 22, n. 14, p. 2249-2261, 2011.

WENDLING, W. T.; EMERENCIANO, D. B.; HOSOKAWA, R. T. Ajuste da função de distribuição diamétrica Weibull por planilha eletrônica. **Floresta**, Curitiba, v. 41, n. 2, p. 205-220, 2011. DOI: http://dx.doi.org/10.5380/rf.v41i2.21869.