Southern Forests: a Journal of Forest Science

ISSN: 2070-2620 (Print) 2070-2639 (Online) Journal homepage: www.tandfonline.com/journals/tsfs20

Height–diameter allometric models for commercial species in the south-western region of the Brazilian Amazon

Lucas Dalmolin Ciarnoschi, Sylvio Péllico Netto, Allan Libanio Pelissari, Marcus Vinicio Neves d'Oliveira, Alexandre Behling, Claiton Nardini & João Paulo Sardo Madi

To cite this article: Lucas Dalmolin Ciarnoschi, Sylvio Péllico Netto, Allan Libanio Pelissari, Marcus Vinicio Neves d'Oliveira, Alexandre Behling, Claiton Nardini & João Paulo Sardo Madi (2025) Height–diameter allometric models for commercial species in the south-western region of the Brazilian Amazon, Southern Forests: a Journal of Forest Science, 87:3, 100-111, DOI: 10.2989/20702620.2024.2363752

To link to this article: https://doi.org/10.2989/20702620.2024.2363752

Published online: 21 Jul 2025.

Southern Forests 2025, 87(3): 100–111
Printed in South Africa — All rights reserved

SOUTHERN FORESTS
ISSN 2070-2620 EISSN 2070-2639
https://doi.org/10.2989/20702620.2024.2363752

Height–diameter allometric models for commercial species in the south-western region of the Brazilian Amazon

Lucas Dalmolin Ciarnoschi[1], Sylvio Péllico Netto[1], Allan Libanio Pelissari[1], Marcus Vinicio Neves d'Oliveira[2], Alexandre Behling[1], Claiton Nardini[1]* and João Paulo Sardo Madi[1]

[1] *Department of Forest Sciences, Federal University of Paraná, Curitiba, PR, Brazil*
[2] *Embrapa Acre, Rodovia Rio Branco, Acre, Brazil*
* *Correspondence: claitonardini@gmail.com*

Accurate total height estimates in Brazilian Amazon forests are still scarce. This study aimed to (1) evaluate a method for stratifying data by potential height (quasi-asymptotic heights) to group the species; (2) fit a regression model using potential heights (quasi-asymptotic heights); and (3) apply the traditional method of non-linear regression analysis for fitting an equation. The Näslund model was used to estimate the total height and fitted to a sample of 213 trees from 33 species from the Brazilian Amazon. The deterministic fitting method, which assumes the mean potential height (quasi-asymptotic heights) of each group, proved to be more consistent when compared with the estimates obtained by nonlinear least-squares (traditional method). The increased precision was also achieved by applying the potential height (quasi-asymptotic heights) grouping method. The equations fitted according to both strategies, when applied to the tree groups, yielded good fits and were supported by validation statistics. The estimate from the deterministic method is the most suitable and recommended.

Keywords: deterministic method, height–diameter relationship, non-linear least squares, potential height

Introduction

Linear relationships between allometric variables in planted and native forest stands are rare, while non-linear relationships are more commonly found (Payandeh 1983; Thomas 1996; Feldpausch et al. 2012; Kearsley et al. 2013; Duan et al. 2018). However, a regression analysis, whether linear or non-linear, expresses a functional relationship between a dependent variable and one or more independent variables, and through these relationships, some tree variables (such as diameter, height, volume, and cross-sectional area) can be estimated (Pretzsch 2010).

Non-linear relationships between variables can be transformed into a linear form to apply linear regression analysis (Payandeh 1983; Vanclay 1994; Porté and Bartelink 2002; Pretzsch 2010). However, such transformations can cause changes in the measured variables and in their relationships, which can result in distortions when the regression estimates are retransformed to the original scale (Pretzsch 2010).

Models of height (h) and diameter at breast height (DBH) relationships are useful tools to improve accuracy for estimating above ground biomass and carbon stock in tropical forests (Chave et al. 2005; Arias-Rodil et al. 2017; Kearsley et al. 2017). These relationships have been studied by many researchers (see, for example, Thomas 1996; Nogueira et al. 2008a; Khadka et al. 2015; Vibrans et al. 2015).

The total height data (h_t) in native forests are complex biometric measurements, due to the high operational and financial costs needed to obtain them, especially in tropical forests (Feldpausch et al. 2012; Hunter et al. 2013; Larjavaara and Muller-Landau 2013). As the diameter measurement can be obtained at a lower cost with greater accuracy than height measurements, and there is a close relationship between these two variables, height-diameter equations can be used to estimate heights (Prodan 1968).

According to Thomas (1996), potential height (h_p) is a suitable measure to quantify and group the height variability of trees as a continuous variable. However, the relationship between (h_t) and DBH is influenced by several stand characteristics, such as: (1) site (Trorey 1932; Curtis 1967; Huang et al. 2000; Saunders and Wagner, 2008); (2) soil properties (Heineman et al. 2011; Urban et al. 2013; Aiba and Kitayama 1999); (3) stand development (Curtis 1967; Holbrook and Putz, 1989; Kohyama et al. 1990; Henry and Aarssen 1999; Sterck and Bongers 1998; Poorter et al. 2003), and 4) species composition (Ter Steege et al. 2013; Cardoso et al. 2017).

According to Zimmermann (1983), some allometric relationships tend to become more evident during the plant's development, while others show asymptotic behaviour in the plant's ontogeny. The use of a function that represents a sigmoid curve can be used to estimate the height development of tropical trees (Niklas 1994). However, data on the height growth of forests in the tropics are scarce (Thomas 1996). Thus, a deterministic method to obtain the model's parameters using potential or quasi-asymptotic heights is a feasible solution and was tried experimentally by the authors of this research paper.

Accurate total height estimations for tropical forests are still scarce, especially in the Brazilian Amazon rainforest. This study aimed to: (1) evaluate a method for stratifying data by grouping ecologically potential total heights; (2) fit a regression model using potential heights (quasi-asymptotic heights) to obtain an estimator per commercial species group, and (3) apply the traditional method of non-linear regression analysis to describe the height–diameter relationship. For this, two hypotheses were formulated: H1 – species grouping enables more accurate total height estimates per tree; and H2 – a height–diameter equation fitted using a deterministic method is viable to estimate the total heights of commercial trees in the Amazon forest.

Material and methods

Study area

The database was collected in the Antimary State Forest (FEA), in the south-west region of the western Brazilian Amazon (68°23′ W; 9°13′;9°31′ S). A sub-area of 483.72 ha was selected for exploration in 2017, in which a digital forest exploration model published by Figueiredo et al. (2007) was applied.

Sampling procedure

The sub-area of 483.72 ha chosen for forestry exploration was defined as the population to be sampled. As is traditionally done in these areas, the following steps are specified for sampling: (1) the company previously defines the species to be explored, consequently only trees of these species are included in the sampling; and (2) all trees of these species throughout the area to be explored without defects for commercialisation and with DBH ≥ 50 cm are sampled, except those that present bole characteristics with defects (hollow boles and boles with significant tortuosity).

Based on the census carried out in the sub-area, 213 trees were selected for cutting, using 2 sensitive post-processed Techgear Zenith II L1 GPS receivers. The trees that were used to collect the total height (h_t) through LiDAR (light detection and ranging) (Ferraz et al. 2004) had an accuracy of ($\varepsilon_i \leq 15$ cm) in their position. The collection was performed manually using FUSION/LDV software. In addition to the total height, DBH taken at 1.30 m above ground (d) or above tree buttresses (dr) and commercial height (h_c) were also measured. The tree species were identified as well.

Analysis of forest structure

All measured trees that were qualified for sampling were grouped into four potential height (h_p) classes, based on the tallest height recorded in the literature for commercially exploited species (Loureiro et al. 1979; Lorenzi 1992; Lorenzi 1998, 2002; Souza et al. 2005; Rozendaal et al. 2006; Lorenzi and Matos 2008), in which the following classes were established: (1) > 48.5 m; (2) 48.5–45.5 m; (3) 45.5–42.5 m; and (4) ≤ 42.5 m.

Statistical modelling of tree variables

Descriptive statistics for the variables DBH and h_t were provided using the mean, standard deviation, variance of the mean, standard error (absolute), relative sampling error, coefficient of variation and number of observations (Péllico Netto and Brena 1997; Araújo et al. 2018). Additionally, the confidence intervals were calculated at 95% probability level.

To test hypothesis 2, the authors considered the statement by Thomas (1996) about the use of potential height to represent the hypsometry of species or groups of species, as well as the distinctive characteristics of the Näslund (1936) model. The authors concluded that only this model could be used for adjustment in a deterministic way, as it facilitates obtaining the quasi-asymptotic height and the inflection point of the curve, which are the focus of our work.

Therefore, total height was modelled using the Näslund (1936) model as presented in Equation 1. This model is especially attractive because of its simplicity and ability to describe a sigmoid curve representative of biological and ecological behaviour of the tropical species.

$$h_t - 1.30 = \frac{DBH^2}{\left(a + bd\right)^2} + \varepsilon_i \quad (1)$$

where h_t is the total height (m); d is DBH at 1.30 m above ground (cm); a and b are coefficients of the model; and ε_i is the estimated random error.

The model was fitted using two strategies: in strategy I, it was deterministically fitted using the methodology described by Näslund (1936), in which the asymptotic values were based on the tallest height recorded in the literature for commercially exploited species. The methodology was conceived as follows:

Taking the limit of Equation 1 when DBH tends to infinity, the asymptote is obtained in Equation 2:

$$\text{Asymptote } (h_p) = \frac{1}{b^2} + 1.3 \quad (2)$$

The coefficient b is obtained by solving Equation 2 and is presented in Equation 5.

$$\frac{1}{b^2} = h_p - 1.30 \quad (3)$$

$$b^2 = \frac{1}{h_p - 1.30} \quad (4)$$

$$b = \sqrt{\frac{1}{h_p - 1.30}} \quad (5)$$

Taking the second derivative of the function (Equation 1) and setting it to 0, we obtained the inflection point on the curve; the height at this point (h_{inf}) is presented in Equation 6, and the diameter at this point ($dinf$) is presented in Equation 7:

$$h_{\text{inf}} = \frac{1}{9b^2} \quad (6)$$

$$d_{\text{inf}} = \frac{a}{2b} \quad (7)$$

Péllico Netto et al. (2015) used the Näslund (1936) model to fit the height–diameter relationship, from which they obtained the inflection points and the asymptote for black wattle (*Acacia mearnsii* De Wild) in Frederico Westphalen, Rio Grande do Sul, Brazil. A similar experiment was also carried out in plantations of *Eucalyptus grandis* Hill, *Mimosa scabrella* Benth. and *Ateleia glazioviana* Baill. at the same location, where growth was evaluated every four months to detect the inflection points that normally occur before two years of age (Bamberg, 2014). Péllico Netto et al. (2015) have taken these inflection points (*A. mearnsii*: d_{inf} = 3.80 cm, h_{inf} =

4.60 m; *M. scabrella*: d_{inf} = 3.0 cm, h_{inf} = 4.29 m; *E. grandis*: d_{inf} = 6.00 cm, h_{inf} = 8.58=m; *A. glazioviana*: d_{inf} = 2.80 cm, h_{inf} = 3.58 cm), and considered that the ratios of

$$\frac{h_{inf}}{d_{inf}}$$

obtained for these four species (*A. mearnsii* = 1.21, *M. scabrella* = 1.43, *E. grandis* = 1.43 and *A. glazioviana* = 1.28) could be used to calculate an average they have called as a calibration value to be used in Equation 7 to estimate the coefficient '*a*' for each species. The value of 1.34 was the result of the average ratio from the experiment. Considering a new species for which the asymptote (h_p) is equal to 31 m, we have:

$$h_p = \frac{1}{b^2} + 1.30$$

and thus:

$$\frac{1}{b^2} = h_p - 1.30$$

By isolating and equating for *b*, we get:

$$b = \sqrt{\frac{1}{31-1.30}} = \sqrt{0.03367} = 0.1835$$

Substituting the value of *b* into (6) we can obtain the height of the inflection point h_{inf} as follows:

$$h_{inf} = \frac{1}{9b^2} = \frac{1}{9(0.1835)^2} = 3.2998$$

By substituting h_{inf} and d_{inf} into the ratio we can solve for d_{inf} in Equation 8 as follows:

$$\frac{\frac{1}{9b^2}}{\frac{a}{2b}} = \frac{0.2222}{a(0.1835)} 1.34 \qquad (8)$$

$$\text{and therefore } a = \frac{0.2222}{1.34(0.1835)} = 0.9037$$

Finally, substituting the estimated value of 'a' into (7) we can obtain d_{inf} or, likewise, substituting h_{inf} into (8). The adjustment for this species in a deterministic way can be made using the obtained values (asymptote h_p = 31 m, height of the inflection point h_{inf} = 3.2998 m, diameter of the inflection point d_{inf} = 2.4625 cm). The coefficient *a* is obtained in Equation 9:

$$a = DBH_{\text{inf}} \times 2b \qquad (9)$$

Using the *a* and *b* values, the replacement can be made in the expanded form of Equation 1 and simplified in Equation 10.

$$h_t - 1.30 = \frac{d^2}{a^2 + 2bad + b^2d^2} \qquad (10)$$

Replacing: $a^2 = \beta_0$; $2ba = \beta_1$; e $b^2 = \beta_2$, we have:

$$h_t - 1.30 = \frac{d^2}{\beta_0 + \beta_1 d + \beta_2 d^2} \qquad (11)$$

By isolating the variable h_t, the final model for estimating heights is obtained:

$$h_t = \frac{d^2}{\beta_0 + \beta_1 + \beta_2 d^2} + 1.30 \qquad (12)$$

In strategy II, the Näslund(1936) model was fitted using the non-linear least squares (NLLS) method, as shown in Equation 13:

$$y_i = f(x_i, \theta) + \varepsilon_i (i = 1, ..., n) \qquad (13)$$

where y_i is h_t; $f(x_i, \theta)$ is a non-linear continuous function, with known form (1), of the vector of explanatory variables $x_i(d)$ and unknown parameters θ (β $\theta(\beta_0$, β_1 e, $\beta_2)$; and ε_i is the random error.

In Equation 13, the NLLS estimate of θ, denoted by $\hat{\theta}$, minimises the sum of squares of errors (*SQE*) over $\theta \in \Theta$.

$$SQE(\theta) = \sum_{i=1}^{n} [y_i - f(x_i, \theta)]^2 \qquad (14)$$

To estimate non-linear least squares (NLLS), Equation 14 must be differentiated with respect to each parameter, and the resulting equations set to zero to solve for each parameter.

$$\frac{\partial SQE(\theta)}{\partial \theta_r}\Big|_{\hat{\theta}} = 0 (r = 1, 2, ..., p) \sum_{i=1}^{n} [y_i - f(x_i, \theta)]$$
$$\frac{\partial (fx_i, \theta)}{\partial \theta_r}\Big|_{\theta = \hat{\theta}} = 0 (r = 1, 2, ..., p) \qquad (15)$$

A system of normal equations for the Näslund (1936) function is determined. This system of normal equations cannot be solved analytically, so the application of iterative methods was necessary. The Gauss–Newton algorithm was used, in which values for the input parameters were obtained through strategy I.

Suppose that $\theta^{(a)}$ approximates the NLLS estimate ($\in$) of a non-linear model and (*a*) is the iteration index. For θ close to $\theta^{(a)}$, an expansion in the first-order Taylor series was considered $f(x, \theta) \approx f(x, \theta^{(a)}) + F(\theta^{(a)})(\theta - \theta^{(a)})$, where

$$F(\theta) = \frac{\partial f(x, \theta)}{\partial \theta'} = [\frac{\partial f(x_i, \theta)}{\partial \theta_j}].$$

Defining $r\theta$ as a vector of the residuals, we can write:

$$r(\theta) = y - f(x, \theta) \approx r(\theta^{(a)}) - F^{(a)}(\theta - \theta^{(a)})$$

where $F(a) = F(\theta^{(a)})$. Replacing $r'(\theta)r(\theta)$ in $SQE(\theta)$, we get the equation:

$$SQE(\theta) \approx r'(\theta^{(a)})r(\theta^{(a)}) - 2r'(\theta^{(a)})F^{(a)}(\theta - \theta^{(a)}) + (\theta - \theta^{(a)})' F'^{(a)} F^{(a)} (\theta - \theta^{(a)}) \qquad (16)$$

Therefore, Equation 16 is minimised when $\theta - \theta^{(a)} = [F'^{(a)} F^{(a)}]^{-1} F'^{(a)} r[\theta^{(a)}]$.

With the approximation of $\theta^{(a)}$, the next term is given by:

$$\theta^{(a+1)} = \theta^{(a)} + [F'^{(a)} F^{(a)}]^{-1} F'^{(a)} r[\theta^{(a)}] \qquad (17)$$

This results in an iterative process which is repeated until convergence is obtained.

The quality of the fitting was assessed using the following statistics: pseudo-coefficient of determination (r^2), which is given by the square of Pearson's linear correlation coefficient (*r*) between observed and estimated values; standard error of the estimate, (s_{yx}%); graphical analysis of standardised residuals ($R_{pad.}$); significance of the regression coefficients, using Student's *t*-test at the 95% probability level; and the Akaike Information Criterion (AIC). In addition, the chi-square (χ^2) test was applied at the 95% probability level to assess adherence between the estimates obtained by the two proposed strategies.

With the result of the χ^2 test not significant for all combinations, it was possible to obtain the validation statistics using bias (Equation 18), sum of square of the relative error (Equation 19), and percentage of residuals (Equation 20), according to the methodology used by Figueiredo-Filho et al. (1996), Bragg (2008), Souza et al. (2008), Costa et al. (2014), Temesgen et al. (2014), Mensah et al. (2018), and Péllico Netto and Behling (2019), whose selection of the most suitable strategy for estimating the total

Table 1: Descriptive statistics of the variables diameter at breast height (DBH) and height values (h_t) for general data and by potential height classes in species with commercial interest in the south-western region of Brazilian Amazon

Variable	Class	$\bar{x}$	CV *(%)*	*s*	$s^2_{\bar{x}}$	$S_{\bar{x}}$	E_r	*n*
DBH (cm)	General	91.09	29.12	26.53	1.65	1.29	2.33	221
	1	107.35	31.99	34.34	32.76	5.72	9.01	38
	2	90.06	26.97	24.29	4.76	2.18	4.01	126
	3	78.44	24.98	19.60	22.59	4.75	10.58	19
	4	84.32	24.95	21.04	12.29	3.51	7.03	38
h_t (m)	General	37.55	11.84	4.45	0.05	0.22	0.95	221
	1	41.62	8.95	3.73	0.39	0.62	2.52	38
	2	38.14	9.00	3.43	0.10	0.31	1.34	126
	3	35.48	8.60	3.05	0.55	0.74	3.64	19
	4	32.39	11.01	3.57	0.35	0.59	3.10	38

$\bar{x}$ = arithmetic mean; *CV (%)* = coefficient of variation; *s* = standard deviation
$s^2_{\bar{x}}$ = variance of the mean; $S_{\bar{x}}$= standard error; E_r = relative sampling error
n = number of observations

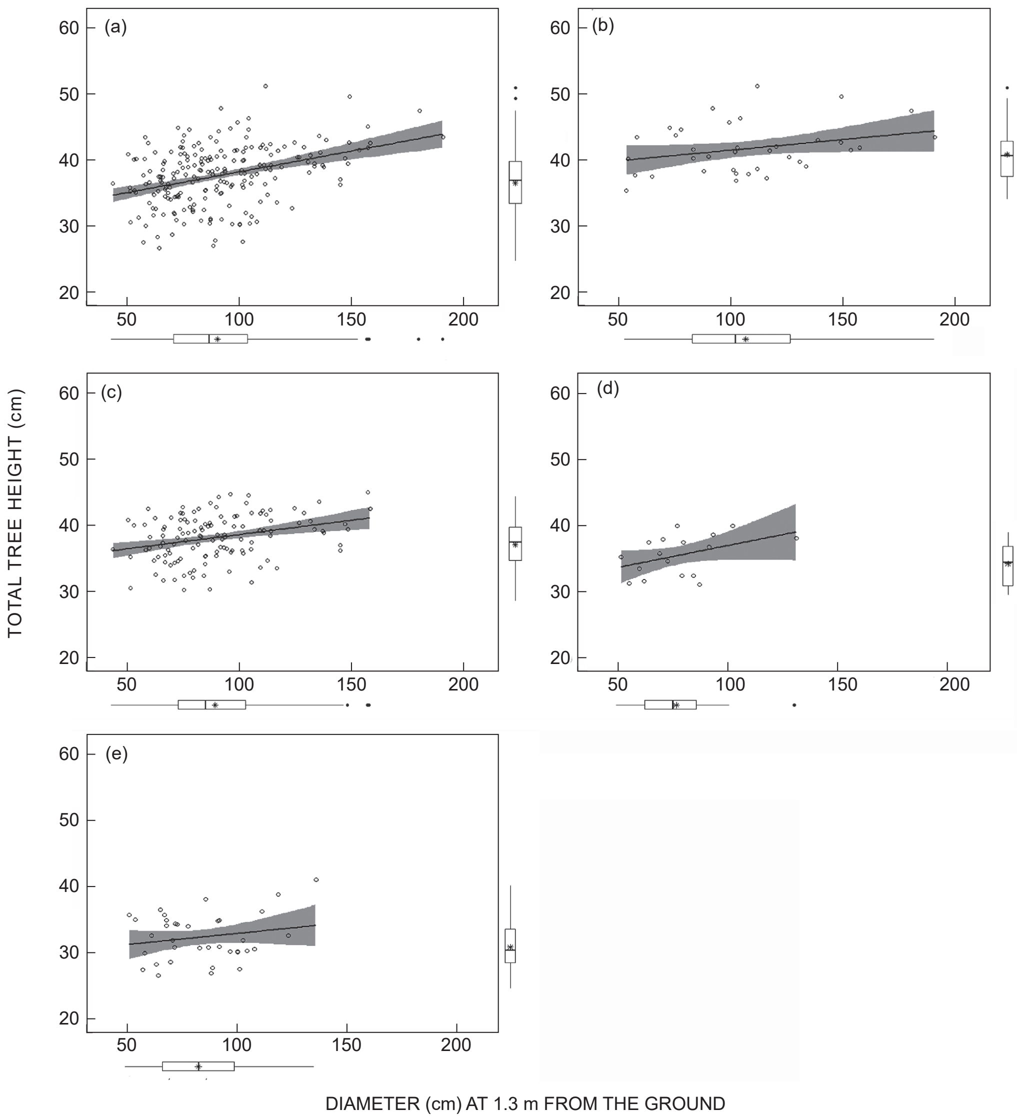

Figure 1: Distribution of height and diameter data with confidence interval on a linear regression line and box-plots with the mean, for species of commercial interest in the southwestern region of Brazilian Amazon. (a) Total data set, (b) Class 1 (h_p > 48.5 m), (c) Class 2 (48.5 ≥ h_p > 45.5 m), DBH) Class 3 (45.5 ≥ h_p > 42.5 m; e e) Class 4 (h_p ≤ 42.5 m)

Table 2: Statistics and coefficients referring to the adjustments of the Näslund (1936) function for height values (h_t), general data and potential height classes, for species with commercial interest in the south-western region of Brazilian Amazon

Estimator	Class	*β0*	*β1*	*β2*	*r²*	S_{yx}	S_{yx}%	AIC
Strategy I	General	1.9315	0.4205	0.0229	0.712	4.2	11.58	139 425.85
	1	2.0602	0.4140	0.0205	0.839	3.9	9.85	3 779.04
	2	1.9828	0.4166	0.0219	0.624	3.4	9.14	38 675.15
	3	1.9061	0.4226	0.0234	0.937	2.9	8.98	685.93
	4	1.7802	0.4346	0.0265	0.769	3.7	12.10	3 649.82
Strategy II	General	1.9182	0.3242	0.0235	0.714	4.1	11.41	138 014.30
	1	3.4729	0.1079	0.0233	0.860	3.6	9.14	3 561.97
	2	2.5006	0.1794	0.0247	0.646	3.3	8.80	37 484.38
	3	2.3812	0.3028	0.0248	0.938	2.9	8.89	678.56
	4	2.9000	0.0679	0.0308	0.784	3.6	11.69	3 550.00

β1 = the model's regression coefficient *r²* = pseudo-*R²* *Syx* = standard error of the estimate in meters
Syx% = standard error of the estimate in percentage AIC = Akaike information criteria

Table 3: Chi-square (χ^2) test to evaluate the adherence of total height estimates between strategies and by potential height classes, for species of commercial interest in the south-western region of Brazilian Amazon

Class	Comparisons	X^2_{cal}
General	Strategy I – Strategy II	1.166 ns
	Strategy I – Observed height	105.549 ns
	Strategy II – Observed height	105.589 ns
1	Strategy I – Strategy II	1.800 ns
	Strategy I – Observed height	11.972 ns
	Strategy II – Observed height	10.275 ns
2	Strategy I – Strategy II	2.873 ns
	Strategy I – Observed height	39.440 ns
	Strategy II – Observed height	36.706 ns
3	Strategy I – Strategy II	0.074 ns
	Strategy I – Observed height	3.615 ns
	Strategy II – Observed height	3.568 ns
4	Strategy I – Strategy II	1.083 ns
	Strategy I – Observed height	14.625 ns
	Strategy II – Observed height	13.598 ns

χ^2_{cal} = calculated chi-square ns = nonsignificant at 95% probability
$\chi^2_{(0.05;221)}$ = 186.671 $\chi^2_{(0.05;38)}$ = 52.192
$\chi^2_{(0.05;126)}$ = 100.178 $\chi^2_{(0.05;19)}$ = 9.390

height was determined by the weighted value (Equation 21); thus, the lowest value will be chosen. The fit and statistics were done using the SAS software (Statistical Analysis System) for academics.

$$Bias = \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)}{N} \quad (18)$$

$$SEQR = \sum_{i=1}^{n}[\frac{(y_i - \hat{y}_i)}{y_i}]^2 \quad (19)$$

$$RP = \frac{[\sum_{i=1}^{n}(\frac{(y_i - \hat{y}_i)}{y_i})]100}{N} \quad (20)$$

$$VP = \sum_{i=1}^{n} Nr_i \times P_i \quad (21)$$

where y_i is the observed variable; $\hat{y}_i$ is the estimated variable; *N* is the number of observations; *SEQR* is the sum of squares of the relative error; *RP* is the residual in percentage; *VP* is the weighted value; Nr_i is the number of observations in the i^{th} placement; and P_i is the weight of the i^{th} placement.

Results

Based on the census data, trees with potential timber production were selected to be cut, their potential heights were searched in the literature, and subsequently they were distributed in four height classes: Class 1: *Dipteryx odorata* (Aubl.) Willd. (Cumaru-ferro), *Ceiba pentandra* (L.) Gaertn. (Samauma branca), and *Couratari oblongifolia* Ducke & Kunth. (Tauari); Class 2: *Pouteria* sp. (Abiurana), *Andira* sp. (Angelim-coco), *Cassia* sp. (Bajão), and *Simarouba amara* Aubl. (Box), *Anacardium giganteum* W.Hancock. ex Engl. (Cajui), *Qualea tessmannii* Mildbr. (Catuaba), *Amburana acreana* (Ducke) A.C.Sm. (Cherry), *Apuleia leiocarpa* (Vogel) J.F.Macbr. (Garapeira), *Barnebydendron riedelii* (Tul.) J.H.Kirkbr. (Guaribeiro), *Clarisia* sp. (Guariuba), *Hymenaea courbaril* L. (Jatoba), *Manikara bidentata subsp surinamensis* (Miq.) T.D.Penn. (Maçaranduba), *Pouteria* sp. (Maparajuba), *Escheweilera coriacea* (DC.) S.A.Mori (Matamata-rosa), *Terminalia* sp. (Mirindiba), Unidentified, *Ceiba samauma* (Mart.) K.Schum. (Samauma vermelha), *Platymiscium trinitatis* Benth. (Violeta) and *S. pruriens* (Aubl.) K.Schum. (Xixa); Class 3: *Aspidosperma vargasii* A. DC. (Amarelão), *A. ruizii* Klotzsch (Ashtray), *Copaifera* sp. (Copaíba-preta), *Handroanthus serratifolius* (Vahl) S.Grose (Ipê-Amarelo), *Astronium lecointei* Ducke (Maracatiara), *Agonandra brasiliensis* Miers ex Benth. & Hook.f. (Ivory), and *Erythrina fusca* Lour. (Mulungu); and Class 4: *Pouteria sp.* (Abiu), *Castilla ulei* Warb. (Caucho), and *Cedrela odorata* L. (Cedro-rosa).

The statistics presented in Table 1 show that the variable h_t presented low standard deviation values, both for the general data and data grouped in the h_p classes. The coefficient of variation (CV%) ranged from 8.60% to 11.84%, while the highest sample error was found in Class 3 (3.64%).

Dispersion measures for *d* were higher than those obtained for h_t, given the greater amplitude of this variable, which is reflected in the higher values of CV% and sampling error (E_r). However, even though large errors are acceptable for

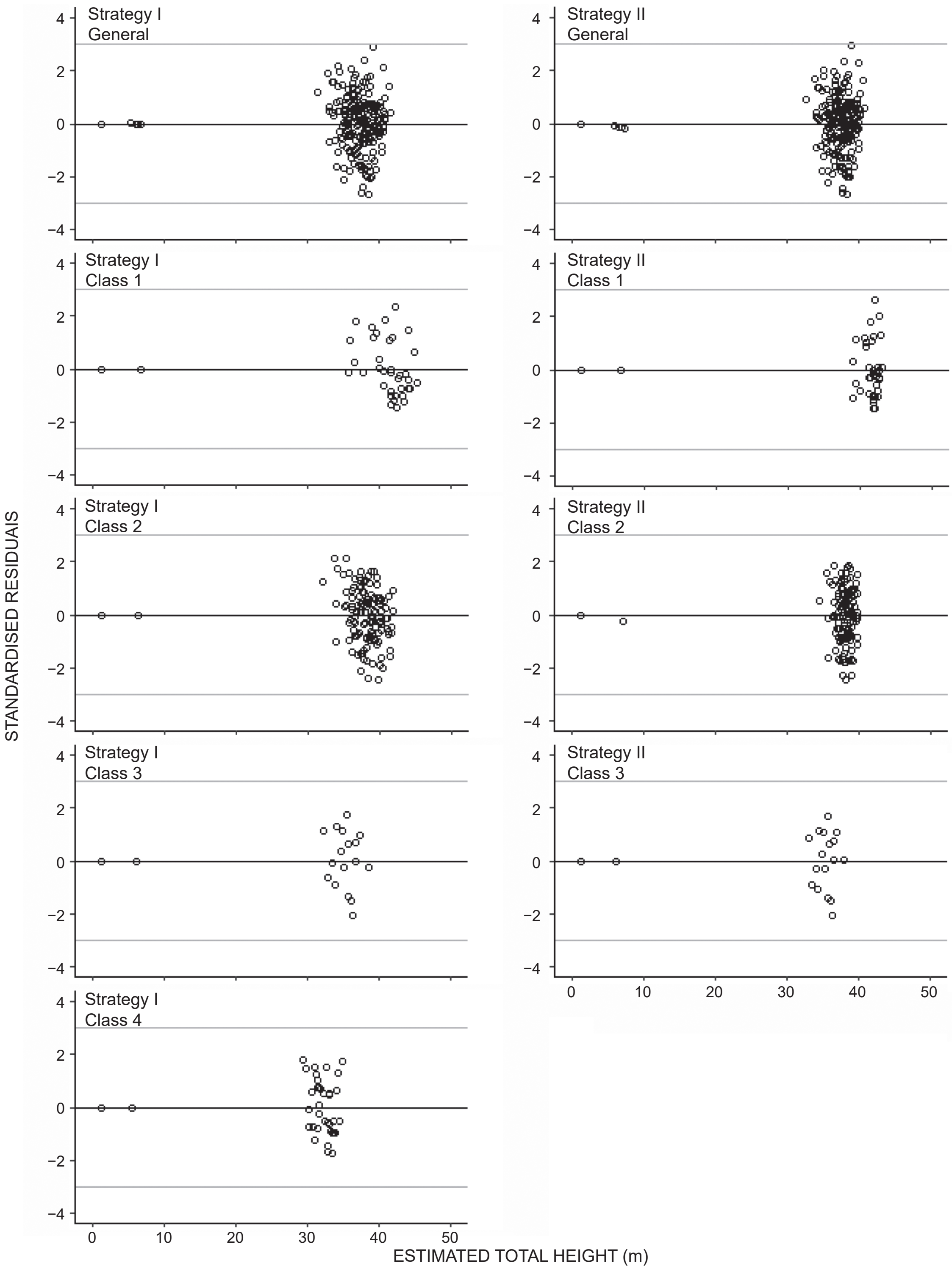

Figure 2: Distribution of normalised residuals of total height estimates by strategies (I and II) and for potential height classes, for species with commercial interest in the southwestern region of Brazilian Amazon

estimates in native forest, in this study the largest error was found in Class 3 (10.58%).

The dispersion data are presented in Figure 1, together with box plots for each data set, which show the data distribution and outliers. The closer the mean and median values are, the greater the chance that the sample data will meet normality. All classes had a symmetrical distribution for the variable h_t, which corresponds to a normal distribution as confirmed by the Shapiro–Wilk test ($p \geq 0.05$).

Class 1 in Figure 1b had the greatest distance between the median and the mean for *d*, resulting in a positive asymmetry, but there was no significant difference between them. In

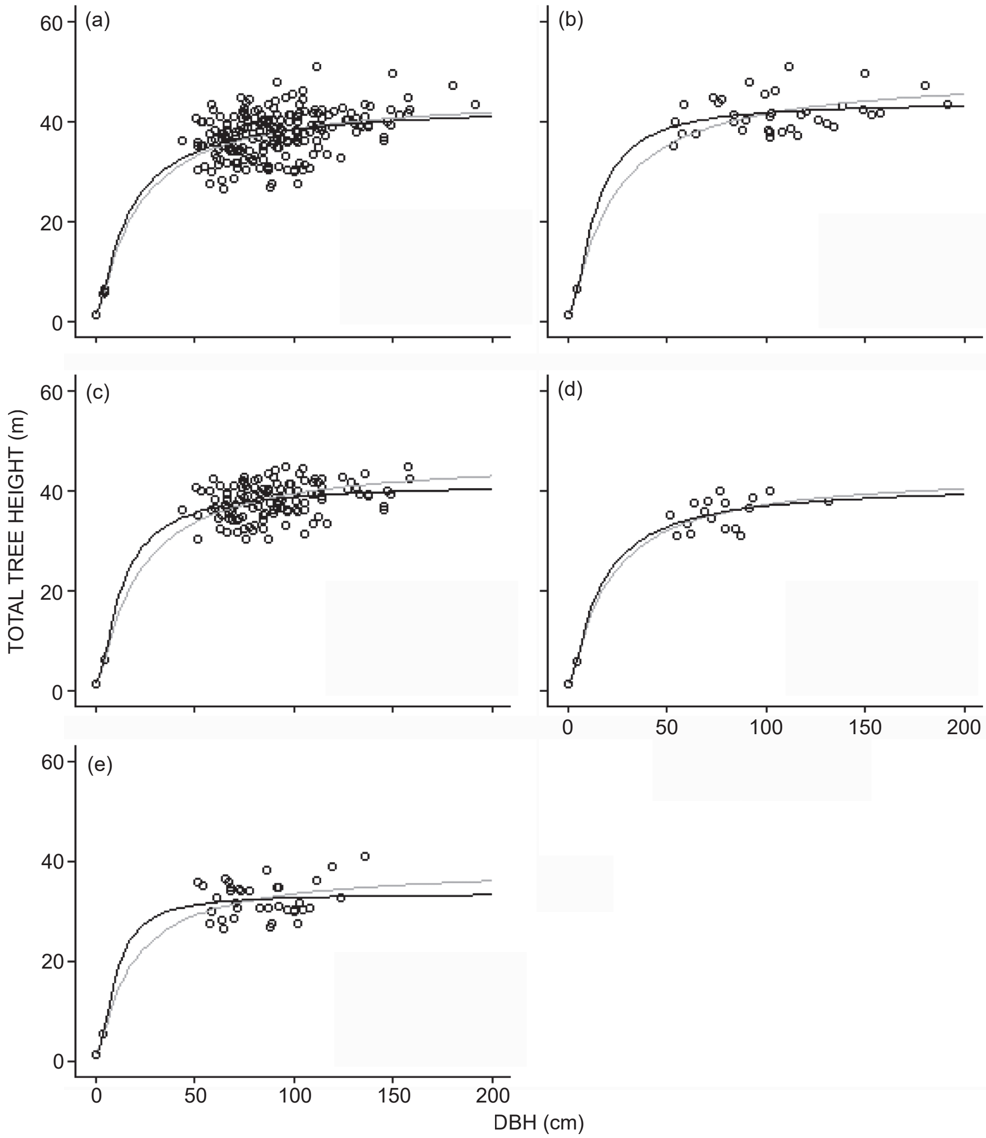

Figure 3: Distribution of height and diameter data with regression curves generated by the Näslund equation (1936) adjusted for species of commercial interest in the southwestern region of Brazilian Amazon

Classes 1, 2 and 3, the data showed normal distributions with a slight tendency toward positive asymmetry, and Class 4 showed a slight negative asymmetry.

The points lying outside the lower limit (1st quartile) and upper limit (3rd quartile) of the box plots for both variables h_t and d, do not necessarily mean a case of outliers, but instead represent values that extend the tail of the normal distribution to the right, that is, outside the range of ±2 698σ.

The regression coefficients and statistical results by strategies I and II for the general data and h_t classes are presented in Table 2. All parameter estimates were significant at 95% probability level. The values of r^2 for all fits were considered high, ranging from 0.624 to 0.938; the s_{yx} (%) were low for both strategies, varying from 8.8% to 12.1%. The AIC values were very high, ranging from 678.56 to 139 425.80.

The normalised residual graphs for the h_t estimates are shown in Figure 2. The limits that are treated as the maximum/minimum (±3σ) were not exceeded in the fittings and were randomly distributed within these limits, which correspond to a normal distribution. In addition, they did not show any tendencies and can be considered homoscedastic, indicating that it is not necessary to transform the data or use weightings.

When comparing the two adjustment strategies, strategy I had a greater amplitude in the estimates, particularly when evaluating by class. However, this did not affect the adjustment statistics. Figure 3 shows the dispersion data with a curve of h_t as a function of d, in which it was possible to evaluate the behaviour of the regression curves together with the dispersion of the real data. Curves were identified to present tendency to have inflection points in different positions when comparing the two adjustment strategies.

When comparing the two adjustment strategies, strategy I showed greater amplitude in the estimates, particularly when evaluating by class. However, this did not affect the adjustment statistics.

In Table 3 the results of χ^2 are presented, where total

Table 4: Complementary statistics to evaluate the precision of strategies for estimating total height, for species of commercial interest in the south-western region of Brazilian Amazon

Estimator	Class	Bias	HEV	SSR	HEV	RP	HEV	WV
Strategy I	General	0.1519	(2)	3.0698	(1)	−0.8097	(1)	4
	1	0.2048	(2)	0.2799	(2)	−0.0715	(1)	5
	2	−0.1122	(2)	1.0978	(2)	−0.9927	(2)	6
	3	0.1420	(2)	0.1056	(2)	−0.0915	(1)	5
	4	0.0579	(2)	0.4602	(2)	−0.8825	(1)	5
Strategy II	General	−0.0055	(1)	3.1250	(2)	−1.4105	(2)	5
	1	0.0010	(1)	0.2406	(1)	−0.6673	(2)	4
	2	−0.0035	(1)	1.0359	(1)	−0.8433	(1)	3
	3	−0.0014	(1)	0.1051	(1)	−0.5614	(2)	4
	4	−0.0064	(1)	0.4312	(1)	−1.2552	(2)	4

Bias = systematic error
HEV = hierarchical evaluation of values
RP = residual in percentage
SSR = sum of squared residuals
WV = weighted value

height estimates for all combinations were not significant in harvested commercial trees in the Antimary State Forest. No significant difference (95% probability) was found between the estimates of the two adjustment strategies for the Näslund (1936) height–diameter model.

Complementary validation statistics are shown in Table 4, which can inform the selection of the best adjustment method and inform the occurrence of possible biases in the estimates.

Discussion

The question of how to obtain exact values for total heights in tropical forests, the most appropriate techniques to be used, and what tools are needed to obtain them, is an important challenge. The method used to group the species by potential height was taken from past research (Rozendaal et al. 2006; Lorenzi and Matos 2008) and replicated in other phyto-physiognomies. Different strategies were used to calculate the model coefficients and the total heights, which could then be evaluated and compared to check for accuracy.

The Näslund (1936) model was selected for this study, despite the concerns highlighted by Fayolle et al. (2016) and Mensah et al. (2018), who emphasised that the use of different models to express the relationship $\frac{h_t}{d}$ can lead to differences in total height estimates. However, this model was chosen because it is a non-linear function, it reflects biological and ecological characteristics in its structure, and because its coefficients can be estimated using different methods, such as a deterministic one that considers the potential height of the species (strategy I) and the NLLS (strategy II).

Other variables (volume, biomass and carbon) are difficult to estimate accurately in tropical forests due to the great diversity of tree species (Chave et al. 2006; Andrade et al. 2017; Mensah et al. 2018; Johnson et al. 2018; Arellano et al. 2019; Rozendaal et al. 2019); and also to high variability in diameters and total heights, because of the phytogeographic characteristics of the forest (Brown et al. 1989). It is costly to measure total height of all trees in a test site. However, we used this option to improve the precision of the estimates.

Nogueira et al. (2008a) posited that there are significant differences in height–diameter relationships in different regions of the Brazilian Amazonia, pointing out that trees in the southern and south-western Amazonia forests have smaller heights at the same diameter than trees located in central Amazonia. The variation of vegetation in the meso-scale, that is, over small geographical distances, and the changes in the tree shape of each species are adaptations to the chemical and ecological conditions of different locations (Rozendaal et al. 2006; Nogueira et al. 2008a). Therefore, hypothesis 1 has been proven, because grouping tree species by potential heights resulted in increased precision in estimating the heights of the species in the study area (Table 2 and Table 4).

An evaluation of the descriptive statistics (Table 1) for d and h_t indicated that they represent well the population sampled, because, as described by Péllico Netto and Brena (1997), a sampling error of less than 10% indicates that the sample of h_t was sufficient to absorb the variation of the variable height. The relative errors found in this research can be considered low because of the high variability of species in the region. Chave et al. (2014), Arias-Rodil et al. (2017), Duan et al. (2018) and Araújo et al. (2018) found similar values to the findings in the test site. The statistics used to assess the fit quality (Table 2) yielded similar values for the two equation fitting strategies. However, the values obtained for the AIC presented a very high amplitude, as described by Fayolle et al. (2016) and Mensah et al. (2018).

The statistics obtained for strategy II, except for the standard error of the estimate of Class 3, were higher than those obtained in strategy I. However, the predictions of the total heights were not significantly affected since the estimates did not differ from each other, as verified in the results of the chi-square test (Table 4).

The results of the fitting statistics were better than those obtained in previous studies carried out in native tropical forests (Fang and Bailey 1998; Chave et al. 2005; Rozendaal et al. 2006; Bragg 2008; Nogueira et al. 2008a; Nogueira et al. 2008b; Feldpausch et al. 2011; Feldpausch et al. 2012; Lima et al. 2012; Hunter et al. 2013; Temesgen et al. 2014; Mensah et al. 2018; Sullivan et al. 2017; Duan et al. 2018). This can be explained by the strategy used of stratifying the data ecologically.

In Figure 3, the regression curves were plotted for both strategies. In strategy I, the regression curve tended to stabilise more smoothly in the asymptotic region, showing the same trend for all classes. This stabilisation can be explained by two factors: (1) the characteristics of the model and the strategy used to make the fitting; and (2) the stratification by groups of species with similar characteristics of height–diameter relationships.

In strategy II, the curves tended to show a more abrupt stabilisation of height development as a function of diameter, occurring earlier than the biological behaviour shown by the sample data (Figure 3b, c, e). This behaviour can be explained: it is due to the regression curve created by strategy II being directly affected by the average value of h_t. This is a characteristic of the fitting method (NLLS), which works to reduce the sum of squares of the residuals without considering the ecological and biological perspective, unlike strategy I.

The estimates obtained in strategy I can be considered acceptable, as they had good statistical fitting results (Table 2). In addition, the chi-square adherence test was not significant (Table 3) for comparing both the actual data and estimates obtained from strategy II. Similar statistical tests were used by Behling (2016) to compare the estimates of two different methods for above ground biomass.

The validation statistics (Table 4) showed a difference in the weightings, which can be explained by the characteristics of each calculated statistic where, in the bias, the arithmetic mean of the errors is calculated at each sample point, and for the statistics sum of squares of the relative error (*SQER*) and residuals in percentage (*RP*) for which the error is relative to the mean.

When the results obtained in this research were compared with the validation statistics reported by other studies (Figueiredo-Filho et al. 1996; Bragg, 2008; Souza et al. 2008; Costa et al. 2014; Mensah et al. 2018), they were similar, though in most of these cited works the estimates were obtained in planted forests. We can affirm that the results of our study are statistically reliable, given the high variability of the data, both in diameter and height, and in variation between species.

As stated by Bredin et al. 2020, the Amazon forest has a great abundance of species with variations in size and shape, which characterises a large spectrum of specific peculiarities.

Companies that exploit wood in the Amazon must apply the sustained forest management (SFM) technique, which should be based on volumetric estimates obtained from accurate and precise measurements to ensure good quality of the wood to be harvested (Leão et al.2021).

As stated by Cysneiros et al. 2017, the variations in sizes and shapes of trees of different species do not support the use of a single volumetric equation or a single average form factor to ensure the quality of the estimates required by the SFM. This has constituted a challenge for companies operating in the Amazon.

The procedures for estimating the volume of wood harvested in managed areas of the Amazon are outlined in Normative Resolution No. 406 (2009) by the National Environmental Council (CONAMA), which allows the use of a uniform form factor of 0.7 for all sizes of trees and species. This overly simplistic approach has led to volume underestimations in smaller trees and overestimations in larger ones, resulting in consistently low accuracy in the estimation of commercial volumes (Cardoso et al. 2024).

The qualitative adjustment of volume equations depends strictly on the variable total or partial height of the trees, which are not always measured adequately in field data collection due to the size of the trees, lack of visibility to carry out measurements, slopes of the terrain and other adverse obstacles encountered, thus contributing to the occurrence of uncertainties and lack of accuracy in these estimates (Cysneiros et al. 2017; Péllico Netto and Behling, 2019; Socha et al. 2020; Leão et al. 2021) .

Cardoso et al. (2024) have stated that current methods used to obtain commercial volumes of trees before harvesting by forest companies have not been sufficiently accurate to ensure the quality of their forest management plans. The authors identified a strong tendency to overestimate the commercial volume in forest inventories, regardless of the occurrence of species and the managed area. The authors highlighted the reasons for this trend: 1) inaccurate measurements of total or partial heights of trees selected for exploration; and 2) use of a single average form factor for all trees and species (0.7), recommended by Brazilian legislation.

The main contribution of this study is to meet a great qualitative demand in obtaining volumetric estimates of all tropical species individually or in groups. With the feasibility of deterministic adjustments using the Näslund (1936) model, it will become possible for researchers in the tropical region to obtain good volumetric estimates of all commercial species to be explored, since estimates of total tree heights of all species will be possible and more reliable.

As presented in the results of this work, it is possible to infer that the height estimates obtained through deterministic adjustment, when compared with those obtained via linear regression adjusted with experimental data, did not present a statistical difference ($\alpha < 0.05$) attest by a χ^2 test. Furthermore, it can be highlighted that for environmental agencies this methodology can also be used to characterise the successional stages of vegetation in a given area, which will enable inspection procedures.

Conclusion

Grouping species by potential height class, known as asymptotes, is an appropriate way to improve the accuracy of the model to estimate total height. It has proved to be an effective criterion for data stratification.

The use of the Näslund (1936) non-linear model resulted in statistically accurate estimates for total height of commercial trees in Brazilian Amazonia.

Strategy I is a promising technique for fitting height–diameter models using a deterministic method. Few studies have been published using this methodology, but we found that it was easy to fit the model, and the results were as good as or better than those obtained by more traditional techniques. However, Strategy II is a commonly used method in forestry and produces consistent estimates with low errors and good fit in regression models.

Grouping of species by potential height classes improved total height estimates. However, as strategy I does not need information on the total height of trees for model fitting, we believe that it is the better technique, as there was no statistically significant difference between the estimates obtained using both strategies.

Acknowledgements — EMBRAPA–Acre is thanked for providing experimental data and logistical support for the collection of additional data at the EMBRAPA Experimental Farm in Acre, Brazil. Lucas Dalmolin Ciarnoschi thanks CAPES for granting the doctoral scholarship that has enabled his research for this publication.

Authorship — LDC collected data, performed modelling work, analysed output data and wrote the first draft of the manuscript, SPN proposed the original idea and revised the manuscript, ALP, MVNO, AB, CN and JPSM contributed substantially to revisions and edited the manuscript.

ORCID iDS

Lucas Dalmolin Ciarnoschi — https://orcid.org/0000-0002-3876-0844
Sylvio Péllico Netto — https://orcid.org/0000-0002-3178-1810
Allan Libanio Pelissari — https://orcid.org/0000-0002-0915-0238
Marcus VN d'Oliveira — https://orcid.org/0000-0001-5593-8897
Alexandre Behling — https://orcid.org/0000-0002-7032-2721
Claiton Nardini — https://orcid.org/0000-0001-5791-6720
João Paulo Sardo Madi — https://orcid.org/0000-0002-9817-2657

References

Aiba SI, Kitayama K. 1999. Structure, composition and species diversity in an altitude-substrate matrix of rain forest tree communities on Mount Kinabalu, Borneo. Plant Ecology, 140: 139–157. https://doi.org/10.1023/A:1009710618040

Andrade RT, Pansini S, Sampaio AF, Ribeiro MS, Cabral GS. 2017. Fitossociologia de uma floresta de terra firme na Amazônia Sul-Ocidental, Rondônia, Brasil. *Biota Amazônia* 7: 36–43. https://doi.org/10.18561/2179-5746/biotaamazonia.v7n2p36-43

Araújo EJGD, Loureiro GH, Sanquetta CR, Sanquetta MNI, Corte APD, et al. 2018. Allometric models to biomass in restoration areas in the Atlantic rain forest. *Floresta e Ambiente 25*. https://doi.org/10.1590/2179-8087.019316

Arellano G, Medina NG, Tan S, Mohamad M, Davies SJ. 2019. Crown damage and the mortality of tropical trees. *New Phytologist* 221: 169–179. https://doi.org/10.1111/nph.15381

Arias-Rodil M, Diéguez-Aranda U, Burkhart, HE. 2017. Effects of measurement error in total tree height and upper-stem diameter on stem volume prediction. *Forest Science* 63: 250–260. https://doi.org/10.5849/FS-2016-087

Bamberg R. 2014. Análise da influência das variáveis meteorológica no crescimento em diâmetro e altura de quatro espécies florestais. Masters dissertation, Federal University of Paraná, Curitiba, Brazil.

Behling A. 2016. Modelagem da biomassa de árvores para assegurar aditividade dos seus componentes, 2016. Doctoral thesis, Federal University of Paraná, Curitiba, Brazil.

Bredin YK, Peres CA, Haugaasen T. 2020. Forest type affects the capacity of Amazonian tree species to store carbon as woody biomass. *Forest Ecology and Management* 473: 118297. https://doi.org/10.1016/j.foreco.2020.118297

Bragg DC. 2008. Comparison of pine height models for the Crossett Experimental Forest. *Journal of the Arkansas Academy of Science* 62: 24–31. https://scholarworks.uark.edu/jaas/vol62/iss1/6

Brown S, Gillespie AJ, Lugo AE. 1989. Biomass estimation methods for tropical forests with applications to forest inventory data. *Forest Science* 35: 881–902. https://doi.org/10.1093/forestscience/35.4.881

Cardoso D, Särkinen T, Alexander S, Amorim AM, Bittrich V, et al. 2017. Amazon plant diversity revealed by a taxonomically verified species list. *Proceedings of the National Academy of Sciences* 114: 10695–10700. https://doi.org/10.1073/pnas.1706756114

Cardoso RM, Miguel EP, de Souza HJ, de Souza ÁN, Nascimento RGM. 2024. Wood volume is overestimated in the Brazilian Amazon: why not use generic volume prediction methods in tropical forest management? *Journal of Environmental Management* 350: 119593. https://doi.org/10.1016/j.jenvman.2023.119593

Chave J, Andalo C, Brown S, Cairns MA, Chambers JQ, et al. 2005. Tree allometry and improved estimation of carbon stocks and balance in tropical forests. *Oecologia* 145: 87–99. https://doi.org/10.1007/s00442-005-0100-x

Chave J, Muller-Landau HC, Baker TR, Easdale TA, Steege HT, Webb CO. 2006. Regional and phylogenetic variation of wood density across 2456 neotropical tree species. *Ecological Applications* 16: 2356–2367. https://doi.org/10.1890/1051-0761(2006)016[2356:RAPVOW]2.0.CO;2

Chave J, Réjou-Méchain M, Búrquez A, Chidumayo E, Colgan MS, et al. 2014. Improved allometric models to estimate the aboveground biomass of tropical trees. *Global Change Biology* 20: 3177–3190. https://doi.org/10.1111/gcb.12629

Costa EA, Finger CAG, Schneider PR, Müller I. 2014. Approximation of numerical integration applied to *Araucaria angustifolia* stem taper models. *Floresta* 45: 31–40. http://dx.doi.org/10.5380/rf.v45i1.32686

Curtis RO. 1967. Height-diameter and height-diameter-age equations for second-growth Douglas-fir. *Forest Science* 13: 365–375. https://doi.org/10.1093/forestscience/13.4.365

Cysneiros VC, Pelissari AL, Machado SD, Figueiredo Filho A, Souza L. 2017. Modelos genéricos e específicos para estimativa do volume comercial em uma floresta sob concessão na Amazônia. *Scientia Forestalis* 45: 295–304. https://doi.org/10.18671/scifor.v45n114.06

Duan G, Gao Z, Wang Q, Fu L. 2018. Comparison of different height–diameter modelling techniques for prediction of site productivity in natural uneven-aged pure stands. *Forests* 9: 63. https://doi.org/10.3390/f9020063

Fang Z, Bailey RL. 1998. Height–diameter models for tropical forests on Hainan Island in southern China. *Forest Ecology and Management* 110: 315–327. https://doi.org/10.1016/S0378-1127(98)00297-7

Fayolle A, Panzou GJL, Drouet T, Swaine MD, Bauwens S, et al. 2016. Taller trees, denser stands and greater biomass in semi-deciduous than in evergreen lowland central African forests. *Forest Ecology and Management* 374: 42–50. https://doi.org/10.1016/j.foreco.2016.04.033

Feldpausch TR, Banin L, Phillips OL, Baker TR, Lewis SL, et al. 2011. Height-diameter allometry of tropical forest trees. *Biogeosciences* 8: 1081–1106. https://doi.org/10.5194/bg-8-1081-2011

Feldpausch TR, Lloyd J, Lewis SL, Brienen RJ, Gloor M, et al. 2012. Tree height integrated into pantropical forest biomass estimates. *Biogeosciences* 9: 3381–3403. https://doi.org/10.5194/bg-9-3381-2012

Ferraz IDK, Leal Filho N, Imakawa AM, Varela VP, Piña-Rodrigues F. 2004. Características básicas para um agrupamento ecológico preliminar de espécies madeireiras da floresta de terra firme da Amazônia Central. *Acta Amazonica* 34:621–633. https://doi.org/10.1590/S0044-59672004000400014

Figueiredo-Filho A, Borders BE, Hitch KL. 1996. Taper equations for *Pinus taeda* plantations in Southern Brazil. *Forest Ecology and Management* 83: 39–46. https://doi.org/10.1016/0378-1127(96)03706-1

Figueiredo EO, Braz EM, d'Oliveira MVN. 2007. Manejo de precisão em florestas tropicais: modelo digital de exploração florestal (2nd edn). Rio Branco: Embrapa.

Heineman KD, Jensen E, Shapland A, Bogenrief B, Tan S, et al. 2011. The effects of belowground resources on aboveground allometric growth in Bornean tree species. *Forest Ecology and Management* 261: 1820–1832. https://doi.org/10.1016/j.foreco.2011.02.005

Henry HAL, Aarssen LW. 1999. The interpretation of stem diameter–height allometry in trees: biomechanical constraints, neighbour effects, or biased regressions? *Ecology Letters* 2: 89–97. https://doi.org/10.1046/j.1461-0248.1999.22054.x

Holbrook NM, Putz FE. 1989. Influence of neighbors on tree form: effects of lateral shade and prevention of sway on the allometry of *Liquidambar styraciflua* (sweet gum). *American Journal of Botany* 76: 1740–1749. https://doi.org/10.1002/j.1537-2197.1989.tb15164.x

Huang S, Price D, Titus SJ. 2000. Development of ecoregion-based height–diameter models for white spruce in boreal forests. *Forest Ecology and Management* 129: 125–141. https://doi.org/10.1016/S0378-1127(99)00151-6

Hunter MO, Keller M, Victoria D, Morton DC. 2013. Tree height and tropical forest biomass estimation. *Biogeosciences* 10: 8385–8399. https://doi.org/10.5194/bg-10-8385-2013

Johnson DJ, Needham J, Xu C, Massoud EC, Davies SJ, et al. 2018. Climate sensitive size-dependent survival in tropical trees. *Nature Ecology & Evolution* 2: 1436–1442. https://doi.org/10.1038/s41559-018-0626-z

Kearsley E, De Haulleville T, Hufkens K, Kidimbu A, Toirambe B, et al. 2013. Conventional tree height–diameter relationships significantly overestimate aboveground carbon stocks in the Central Congo Basin. *Nature Communications* 4: 2269. https://doi.org/10.1038/ncomms3269

Kearsley E, Moonen PC, Hufkens K, Doetterl, S, Lisingo J, et al. 2017. Model performance of tree height-diameter relationships in the central Congo Basin. *Annals of Forest Science* 74: 1–13. https://doi.org/10.1007/s13595-016-0611-0

Khadka A, Subedi T, Ghimire M, Dhakal BP, Parikka H. 2015. Diameter-height models for the Terai tree species. *Banko Janakari* 25: 50–54. https://doi.org/10.3126/banko.v25i1.13471

Kohyama T, Hara T, Tadaki Y. 1990. Patterns of trunk diameter, tree height and crown depth in crowded Abies stands. *Annals of Botany* 65: 567–574. https://doi.org/10.1093/oxfordjournals.aob.a087970

Leão FM, Nascimento RGM, Emmert F, Santos GGA, Caldeira NAM, Miranda IS. 2021. How many trees are necessary to fit an accurate volume model for the Amazon forest? A site-dependent analysis. *Forest Ecology and Management* 480: 118652. https://doi.org/10.1016/j.foreco.2020.118652

Larjavaara M, Muller-Landau HC. 2013. Measuring tree height: a quantitative comparison of two common field methods in a moist tropical forest. *Methods in Ecology and Evolution* 4: 793–801. https://doi.org/10.1111/2041-210X.12071

Lima AJN, Suwa R, de Mello Ribeiro GHP, Kajimoto T, dos Santos J, et al. 2012. Allometric models for estimating above-and below-ground biomass in Amazonian forests at São Gabriel da Cachoeira in the upper Rio Negro, Brazil. *Forest Ecology and Management* 277: 163–172. https://doi.org/10.1016/j.foreco.2012.04.028

Lorenzi H. 1992. Árvores brasileiras: manual de identificação e cultivo de plantas arbóreas nativas do Brasil (vol. 1). São Paulo: Plantarum.

Lorenzi H. 1998. Árvores brasileiras: manual de identificação e cultivo de plantas arbóreas nativas do Brasil (vol. 2). Nova Odessa: Plantarum.

Lorenzi H. 2002. Árvores brasileiras: manual de identificação e cultivo de plantas arbóreas nativas do Brasil (vol. 3). Nova Odessa: Plantarum.

Lorenzi H, Matos FJA. 2008. *Plantas medicinais no Brasil: nativas e exóticas* (ed. 2). Nova Odessa: Plantarum.

Loureiro, AA, Silva MF da, Alencar JC. 1979. *Essências madeireiras da Amazônia*. Vol. II. Manaus: INPA/SUFRAMA.

Mensah S, Pienaar OL, Kunneke A, du Toit B, Seydack A, et al. 2018. Height–diameter allometry in South Africa's indigenous high forests: Assessing generic models performance and function forms. *Forest Ecology and Management* 410: 1–11. https://doi.org/10.1016/j.foreco.2017.12.030

Näslund M. 1936. *Skogsförsöksanstalt ens gallringsförsök i tallskog primär bearbetning* (Meddelanden från Statens skogsförsöksanstalt). Trondheim: Norwegian Forestry Research Institute. https://res.slu.se/id/publ/124955

Niklas KJ. 1994. The allometry of safetyfactors for plant height. *American Journal of Botany* 81: 345–351. https://doi.org/10.1002/j.1537-2197.1994.tb15453.x

Nogueira EM, Fearnside PM, Nelson BW, Barbosa RI, Keizer EWH. 2008a. Estimates of forest biomass in the Brazilian Amazon: new allometric equations and adjustments to biomass from wood-volume inventories. *Forest Ecology and Management* 256: 1853–1867. https://doi.org/10.1016/j.foreco.2008.07.022

Nogueira EM, Nelson BW, Fearnside PM, França MB, de Oliveira ÁCA. 2008b. Tree height in Brazil's 'arc of deforestation': shorter trees in south and southwest Amazonia imply lower biomass. *Forest Ecology and Management* 255: 2963–2972. https://doi.org/10.1016/j.foreco.2008.02.002

Payandeh B. 1983. Some applications of nonlinear regression models in forestry research. *The Forestry Chronicle* 59: 244–248. https://doi.org/10.5558/tfc59244-5

Péllico SN, Behling A. 2019. Additivity of tree biomass components using ratio estimate. *Anais da Academia Brasileira de Ciências* 91. https://doi.org/10.1590/0001-3765201920180272

Péllico Netto S, Brena D. 1997. *Inventário florestal*. Curitiba: Federal University of Paraná.

Péllico Netto S, Sanquetta CR, Caron BO, Behling A, Simon AA, et al. 2015. Ground level photosynthetically active radiation dynamics in stands of *Acacia mearnsii* De Wild. *Anais da Academia Brasileira de Ciências*. 87: 1833–1845. https://doi.org/10.1590/0001-3765201520140080

Poorter L, Bongers F, Sterck FJ, Wöll H. 2003. Architecture of 53 rain forest tree species differing in adult stature and shade tolerance. *Ecology* 84: 602–608. https://doi.org/10.1890/0012-9658(2003)084[0602:AORFTS]2.0.CO;2

Porté A, Bartelink HH. 2002. Modelling mixed forest growth: a review of models for forest management. *Ecological Modelling* 150: 141–188. https://doi.org/10.1016/S0304-3800(01)00476-8

Pretzsch H. 2010. Forest dynamics, growth and yield. Berlin: Springer.

Prodan M. 1968. *Forest biometrics*. Munich: Elsevier.

Rozendaal DMA, Hurtado VH, Poorter L. 2006. Plasticity in leaf traits of 38 tropical tree species in response to light; relationships with light demand and adult stature. *Functional Ecology* 20: 207–216. https://www.jstor.org/stable/3806552

Rozendaal DM, Bongers F, Aide TM, Alvarez-Dávila E, Ascarrunz N, et al. 2019. *Biodiversity recovery of Neotropical secondary forests. Science Advances* 5: eaau3114. https://doi.org/10.1126/sciadv.aau3114

Saunders MR, Wagner RG. 2008. Height–diameter models with random coefficients and site variables for tree species of Central Maine. *Annals of Forest Science* 65: 1–10. https://doi.org/10.1051/forest:2007086

Socha J, Netzel P, Cywicka D. 2020. Stem taper approximation by artificial neural network and a regression set models. *Forests* 11: 79. https://doi.org/10.3390/f11010079

Souza CAMD, Silva GFD, Xavier AC, Mendonça ARD, Almeida AQD. 2008. Avaliação de modelos de afilamento não-segmentados na estimação da altura e volume comercial de *Eucalyptus* sp. *Ciência Florestal* 18: 387–399. https://doi.org/10.5902/19805098450

Souza CR, de Lima RMB, de Azevedo CP, Rossi L. 2005. *Ceiba pentandra*. Manaus: Embrapa Amazônia Ocidental.

Sterck FJ, Bongers F. 1998. Ontogenetic changes in size, allometry, and mechanical design of tropical rain forest trees. *American Journal of Botany* 85: 266–272. https://doi.org/10.2307/2446315

Sullivan FB, Ducey MJ, Orwig DA, Cook B, Palace MW. 2017. Comparison of lidar and allometry-derived canopy height models in an eastern deciduous forest. *Forest Ecology and Management* 406: 83–94. https://doi.org/10.1016/j.foreco.2017.10.005

Temesgen H, Zhang C H, Zhao XH. 2014. Modelling tree height–diameter relationships in multi-species and multi-layered forests: A large observational study from Northeast China. *Forest Ecology and Management* 316: 78–89. https://doi.org/10.1016/j.foreco.2013.07.035

Ter Steege H, Pitman NC, Sabatier D, Baraloto C, Salomão RP, al. 2013. Hyperdominance in the Amazonian tree flora. *Science* 342. https://doi.org/1243092. 10.1126/science.1243092
Thomas SC. 1996. Asymptotic height as a predictor of growth and allometric characteristics in Malaysian rain forest trees. *American Journal of Botany* 83: 556–566. https://doi.org/10.1002/j.1537-2197.1996.tb12739.x
Trorey LG. 1932. A mathematical method for the construction of diameter height curves based on site. *The Forestry Chronicle* 8: 121–132. https://pubs.cif-ifc.org/doi/pdf/10.5558/tfc8121-2
Urban J, Holušová K, Menšík L, Čermák J, Kantor P. 2013. Tree allometry of Douglas fir and Norway spruce on a nutrient-poor and a nutrient-rich site. *Trees* 27: 97–110. https://doi.org/10.1007/s00468-012-0771-y
Vanclay JK. 1994. Modelling forest growth and yield: applications to mixed tropical forests. CABI International.
Vibrans AC, Moser P, Oliveira LZ, Maçaneiro JPD. 2015. Height–diameter models for three subtropical forest types in southern Brazil. *Ciência e agrotecnologia* 39: 205–215. https://doi.org/10.1590/S1413-70542015000300001
Zimmermann MH. 1983. *Xylem structure and the ascent of sap*. Berlin: Springer.