Irrig Sci (1996) 16: 159–167

ORIGINAL PAPER

Dani Or · Henoque R. Silva

Prediction of surface irrigation advance using soil intake properties

Received: 31 July 1995

Abstract A simple method for predicting surface irrigation advance trajectories using infiltration parameters and inflow rate as inputs was developed. The difference between the inflow rate and the sum of infiltration rates over the wetted portion of the field equals the flow rate available for advance. An average (characteristic) infiltration rate ahead of the wet portion is computed using a fixed time step. An advance step (for a fixed time step) is calculated from the ratio of the flow rate available for advance and characteristic infiltration rate. Predictions of advance by the proposed method were compared with field observations, with the kinematic wave model, and with analytical solutions of Philip and Farrell (1964). In all cases, the method provided predictions that were in good agreement with field observations, and performed similarly to the kinematic wave model. The method offers a simple and efficient tool for prediction and evaluation of surface irrigation systems under various soil types and variable inflow rates. The method is particularly useful for predictions in fields with spatially and temporally variable intake properties.

Key words Surface irrigation · Surge flow · Intake properties · Heterogeneous soils

Introduction

The performance of surface irrigation systems is strongly influenced by nearsurface soil hydraulic properties, especially those affecting water intake or infiltration rates. An important characteristic of surface irrigation is the overland conveyance of irrigation water during the advance phase from the inlet to the end of a field. Ideally, shorter advance times result in more uniform irrigation application, due to a smaller spread in intake opportunity times across the field. In practice, however, the advance phase may be long, due to constraints of supply system or soil properties (e. g., high infiltration rates), which result in differences in intake times and in application depths. These make prediction of advance times one of the most important factors in determining the performance of surface irrigation (e. g., uniformity and efficiency) during an irrigation event.

Several models are available for predicting advance times, most of which are based on numerical solutions of overland flow models (e. g., kinematic wave, zeroinertia, etc.), or a detailed volume balance. A review of most available models may be found in Walker and Skogerboe (1987) and Shayya et al. (1993). There are also a few analytical equations for describing the advance phase (Lewis and Milne 1938; Hall 1956; Philip and Farrell 1964) and other phases (Strelkoff 1977). The potential usefulness of analytical solutions for solving practical problems may have been overlooked by practitioners. The computational burden and skills, and the large number of parameters required by most numerical models are not always warranted, especially when considering the large uncertainties in field-determined parameters. For many practical problems a reasonably-accurate prediction of the advance phase provides most of the information needed for design and management.

The objectives of this study were: (i) to develop and test a simple method for obtaining predictions of advance from intake parameters and inflow information only; (ii) to evaluate the usefulness of analytical solutions for advance which require intake information only; and (iii) to propose a scheme for updating infiltration parameters with changes in initial and boundary conditions or soil properties.

Dani Or (✉) · Henoque R. Silva
Department of Plants, Soils and Biometeorology and Department of Biological and Irrigation Engineering, Utah State University, Logan, UT 84322-4820, USA

Theoretical considerations

Prediction of advance rates

The advance rate during an irrigation event is influenced by: the inflow rate, surface slope and roughness, hydraulic cross section, and soil infiltration properties. Focusing on typical surface-irrigated fields having mild slope and relatively smooth surfaces, the soil infiltration rate (i $[LT^{-1}]$), and inflow rate per unit of width (q_{in} $[L^2T^{-1}]$), are the most important factors in determining advance rates (Walker and Skogerboe 1987). The approach we propose for predicting advance rates is based on these two attributes as the only inputs, and neglects surface storage and other parameters.

We focus on furrow irrigation with a constant inflow rate at the inlet, however, the discussion applied to other surface irrigation systems. The proposed approach is based on equating the difference between the inflow rate and the convolution-integral of infiltration rates over the furrow's wetted portion to the "runoff" rate (q_{adv} $[L^2T^{-1}]$) available for advance into the dry portion. An average infiltration rate, (i_{avg}) ahead of wetting (on the dry portion) for a fixed time step, Δt, is computed according to:

$$i_{avg}(\Delta t) = \frac{\int_0^{\Delta t} i\,dt}{\Delta t}. \tag{1}$$

The advance step at time t (for a fixed Δt), is computed according to:

$$\Delta_{x_i} = \frac{q_{in} - \int_0^{x(t)} i(x, t - ts)\,dx}{i_{avg}} \tag{2}$$

where x(t) denotes the total length of the wetted portion, i(x,t-ts) is the infiltration rate at each wetted segment of the furrow, and t-ts is the elapsed time since wetting. Each wetted segment is characterized by an advance step Δx_i and by the elapsed wetting time $t_i = (t\text{-}ts)$. In other words, the potential "runoff" rate (for a fixed Δt) satisfies an average infiltration rate (i_{avg}) over the finite length Δx_i only. The length of Δx_i is determined by i_{avg} and by available "runoff" rate. This concept is equivalent to a differential form of Lewis and Milne's (1938) equation ignoring surface storage. Because quantities are expressed as time rates, mass balance violations are relatively small, thereby reducing the error introduced by neglecting surface storage which is now a differential quantity. Note that the proposed method (Eqs. 1 and 2) is independent of any particular form of infiltration equation.

Infiltration parameters under intermittent applications

Infiltration of irrigation water into the soil may be described by several algebraic infiltration equations. The most common for surface irrigation applications is the Kostiakov-Lewis equation, which was originally proposed by Lewis (1937) but is erroneously attributed to Kostiakov 1932 (see a recent discussion by Swartzendruber 1993), is given by:

$$\text{cumulative: } I = kt^a \qquad \text{rate: } i = \frac{dI}{dt} = akt^{a-1} \tag{3}$$

where I is cumulative depth of infiltration (or the volume of water per unit soil surface area), t is elapsed time, **k** and **a** are empirical parameters, and $i = dI/dt$ is the infiltration rate. The shortcomings of this empirical equation are: (1) it disregards different initial water contents; and (2) for long infiltration times it erroneously predicts a zero infiltration rate. The latter problem was fixed by adding the parameter $\mathbf{f_0}$, a parameter representing the infiltration rate after very long elapsed times. The "modified" Kostiakov-Lewis equation is then: $\mathbf{I = kt^a + tf_0}$ and $\mathbf{i = akt^{(a-1)} + f_0}$.

Another infiltration equation with a form similar to the "modified" Kostiakov-Lewis, but with physically-significant parameters, was derived by Philip (1957 a):

$$\text{cumulative: } I = St^{\frac{1}{2}} + At \qquad \text{rate: } i = \frac{1}{2}St^{-\frac{1}{2}} + A \tag{4}$$

where S is sorptivity which is a function of initial (θ_i) and boundary (θ_0) water contents, $S = S(\theta_0, \theta_i)$, and A is a parameter related to saturated hydraulic conductivity, K_s, ($K_s/2 \le A \le 2K_s/3$). Though both equations (3) and (4) may be used in the proposed model, Philip's equation has the advantage that its parameters can be adjusted to changes in initial water contents or in other soil hydraulic properties (Samani and Yitayew 1989; Silva 1995).

A rational basis for adjusting the infiltration parameters becomes a focal point for obtained predictions of advance rates when water is applied intermittently (such as practiced in surge flow irrigation). Walker and Humpherys (1983) presented an interpolation scheme to account for changes in modified Kostiakov-Lewis parameters with the number of surges applied. The limitations of their ad-hoc scheme were acknowledged by Walker and Humpherys, who stressed the need for a more comprehensive and physically-sound approach.

Although the parameters in Philip's (1957 a) equation may be adjusted for different soil types or different initial conditions, such updating requires more information on soil hydraulic properties than is usually available.

Let us first consider the sorptivity, $S(\theta_0, \theta_i)$, and its dependency on initial water content (Philip 1957 b). The sorptivity is directly related to the soil diffusivity, $D(\theta)$, which, in turn, is dependent on unsaturated hydraulic conductivity and soil retentivity (i. e., $D[\theta] = K[h]dh/d\theta$, where h is the soil matric potential). The parametric model of Brooks and Corey (1964) for soil hydraulic properties given as $(\theta\text{-}\theta_r)/(\theta_s\text{-}\theta_r) = (h_w/h)^{\beta}$, and $K(h) = K_s(h_w/h)^{\eta}$; where θ_s and θ_r are saturated and residual water contents, h_w is bubbling (or critical) pressure, and β and η are fitting exponents, yields soil diffusivity as a power function:

$$D(\theta) = D_0(\theta_0 - \theta)^B \tag{5}$$

where $B = (\eta\text{-}\beta\text{-}1)/\beta$ and $D_0 = K_s h_w (\theta_s\text{-}\theta_r)^{-B-1}/\beta$ (Russo and

Bresler 1980). The sorptivity may then be estimated as (Parlange et al. 1993):

$$S^2(\theta_0,\theta_j)=\frac{4D_0(\theta_0-\theta_j)^{B+2}}{3+2B}. \tag{6}$$

The dependency of S on initial water content is particularly important for predictions under intermittent water application (surge flow).

The dependency of parameter A on initial water content is small (Philip 1957 c), and for most practical purposes may be neglected assuming a constant value for $A \approx K_s/2$ (Parlange, 1977). This is not to say that A does not change; however, most of the changes in A (or in "long time" infiltration rates) may be attributed to changes in near-surface soil hydraulic properties, especially changes in the saturated hydraulic conductivity K_s (Samani and Yitayew 1989; Silva 1995). These changes occur mostly during the first irrigation (when soil surface is least stable) due to changes in soil's pore space. The key for predicting such changes in K_s or A is to relate them to changes in other measurable soil physical properties, such as near-surface soil porosity (ϕ). A methodology for obtaining predictions of changes in pore space induced by cycles of wetting and drying was outlined by Silva (1995). These may then be converted to changes in K_s (and the A parameter), using Kozeny-Carman's or Millington and Quirk's formulae (Scheidegger 1957).

Methods

Tests of proposed advance model

Experimental data collected by Elliott (1980) and others were used to compare the proposed method for predicting advance rates, under continuous and intermittent water application. A numerical model developed by Walker and Humpherys (1983) which employs the kinematic wave approach was solved for several case studies (based on parameter availability) and was used for comparisons. The infiltration parameters in all data sets used in this study were in the form of modified Kostiakov-Lewis equation. A short computer program was written to implement Eq. (2) (copies of the program are available upon request).

Philip and Farrell (1962) have proposed analytical solutions to the Lewis-Milne integral equation using several infiltration equations. Two of their solutions, which ignore the surface storage, were chosen for the comparisons. One analytical solution is based on Philip's equation which predicts the length of the wetted portion as:

$$x(t)=\frac{q}{A}\left[1-e^{\frac{4A^2t}{\pi S^2}}\operatorname{erfc}\left(\frac{2At^{\frac{1}{2}}}{\pi^{\frac{1}{2}}S}\right)\right] \tag{7}$$

where q is the inflow rate per unit of width; the other is based on the Kostiakov-Lewis equation:

$$x(t)=\frac{qt^{1-\alpha}}{k\,\Gamma(1+\alpha)\,\Gamma(2-\alpha)} \tag{8}$$

where $\Gamma()$ is the Gamma function. Both solutions were programmed into a worksheet (Quattro-Pro by Borland, Inc.) using built-in functions and requiring minimal computer time. Since infiltration parameters in all data sets used for this study were reported in the modified Kostiakov-Lewis form, a nonlinear regression was employed to estimate an equivalent set of parameters for Philip's and the Kostiakov-Lewis equations. The original parameters of the modified Kostiakov-Lewis equation were used to generate a set of infiltration data to which Philip's and Kostiakov-Lewis equations were fitted. Figure 1 depicts the procedure for Flowell wheel furrow data (Walker and Humpherys 1983).

Advance model – intermittent applications

For intermittent applications (surge flow), use of the proposed updating scheme for infiltration parameters according to changes in initial water content (Eq. 6), was not possible due to lack of data. In order to facilitate comparisons, we opted for a scheme similar to

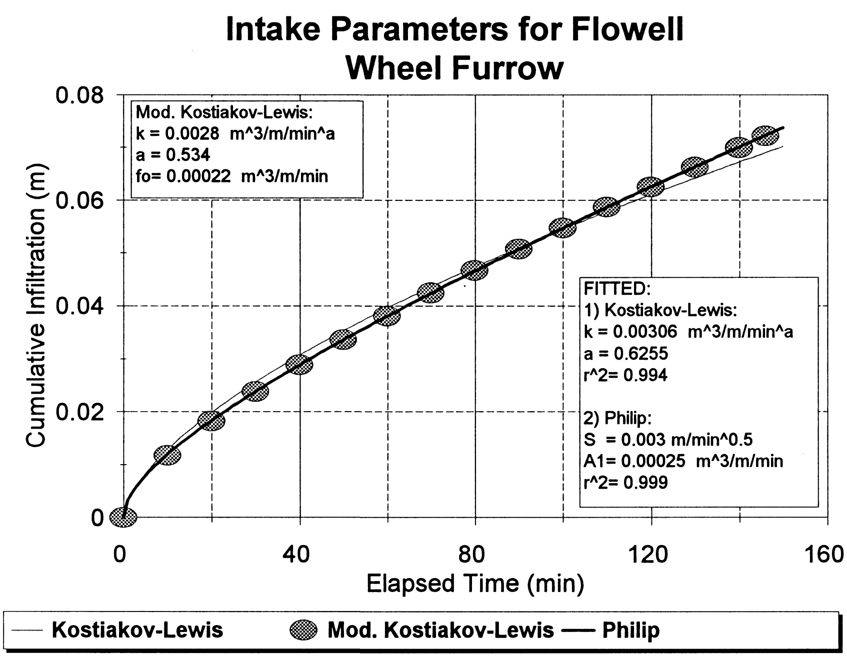


Fig. 1 Estimated infiltration parameters for Philip (Eq. 4) and Kostiakow-Lewis (Eq. 3) equations using modified Kostiakov-Lewis parameters for Flowell wheel-furrow data as input

Walker and Humpherys (1983) to account for variable boundary conditions. The modifications for the intermittent case were as follows: (i) wet and dry sections of a furrow were considered separately, intake parameters for the dry section were identical to the continuous case, whereas "wet" parameters were those measured in subsequent surges; (ii) a constant time-shift equal to the "on" time (i. e., half cycle) was added to the elapsed time for all subsequent surges. The use of a fixed time shift is different from Walker and Humpherys' (1983) scheme which used the total elapsed time from the beginning of the irrigation (first surge).

The proposed modifications are supported by experimental evidence (Silva 1995) showing that the initial water content near the soil surface at the beginning of each subsequent surge was almost identical. An example of changes in water content, matric potential and infiltration rates during intermittent application in a short furrow (2 m) on Millville silt loam is depicted in Fig. 2 (Silva 1995). The repetitive pattern of drying during the "off" phase of subsequent surges justifies a constant, rather than cumulative time "shift". Additionally, very similar final infiltration rates were measured which may be represented by a single set of "wet" parameters for subsequent surges. Finally, we note that the performance of the predictive model was insensitive to the time shift whose main effect was to reduce the highly transient infiltration rates associated with short times (Fig. 2-c).

Results and discussion

Advance rate predictions – continuous flow

Five case studies covering a wide range of soil infiltration parameters and inflow rates were tested. The comparison results for continuous flow are depicted in Fig. 3 (the parameters are given in Table 1). The performance of the proposed method (thick line) in predicting advance rates was satisfactory. It provided advance rate predictions that simulated field data (full circles) as well as the kinematic wave model (connected pluses). The performance of the analytical solution of Philip and Farrell (1964) based on Philip's equation (open circles) performed exceptionally well considering the simplifying assumptions made in its derivation. The Kostiakov-based solution (not shown) performed well for short distances, but in general tended to over-predict advance and thus will not be discussed further.

The exceptional predictability of advance rates based on intake information only, as demonstrated by the newly-proposed method and by Philip and Farrell's solution, emphasizes the dominant role of the infiltration process in surface irrigation. Considering the large spatial variations in soil intake properties (and other uncertainties), detailed numerical modeling may not always be needed, and our proposed simplified method should perform just as well. The main value of the method is in its modest parameter requirements and simplicity, while retaining the main features of the physical process.

Advance rate predictions – intermittent (surge) flow

The performance of the proposed method in predicting advance rates under intermittent applications (surge flow) was tested using three different data (Table 2): (i) Flowell wheel-furrow data (Walker and Humpherys 1983); (ii) Millville silt loam wheel-furrow reported by Coolidge (1981); and (iii) data measured in a non-wheel furrow on red-yellow Oxisol from central Brazil (Oliveira[1] 1989). The main difference between the wheel and no-wheel furrows is in the reduced intake properties in the trafficked

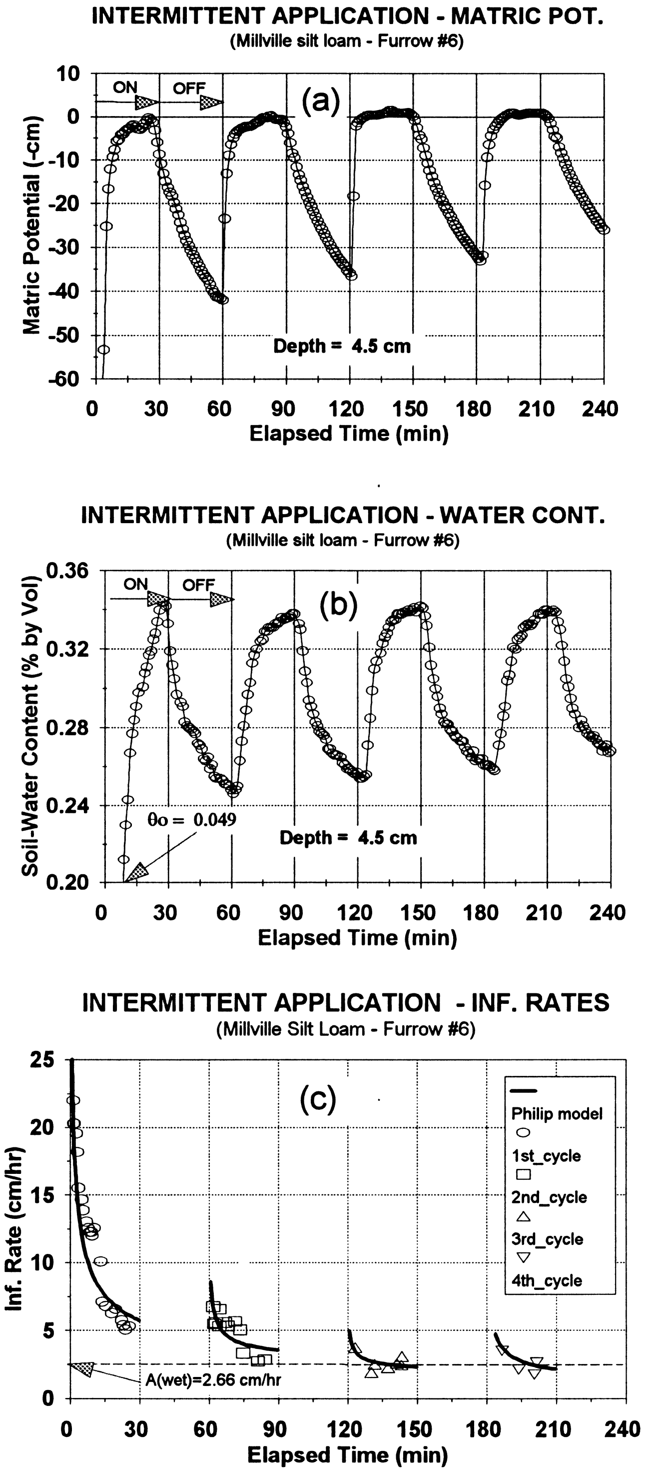


Fig. 2 a–c Temporal changes in near-surface **a** matric potential; **b** water content; and **c** infiltration rates under intermittent application (60 min cycle) in a 2 m furrow on freshly-tilled Millville silt loam soil (Silva 1995)

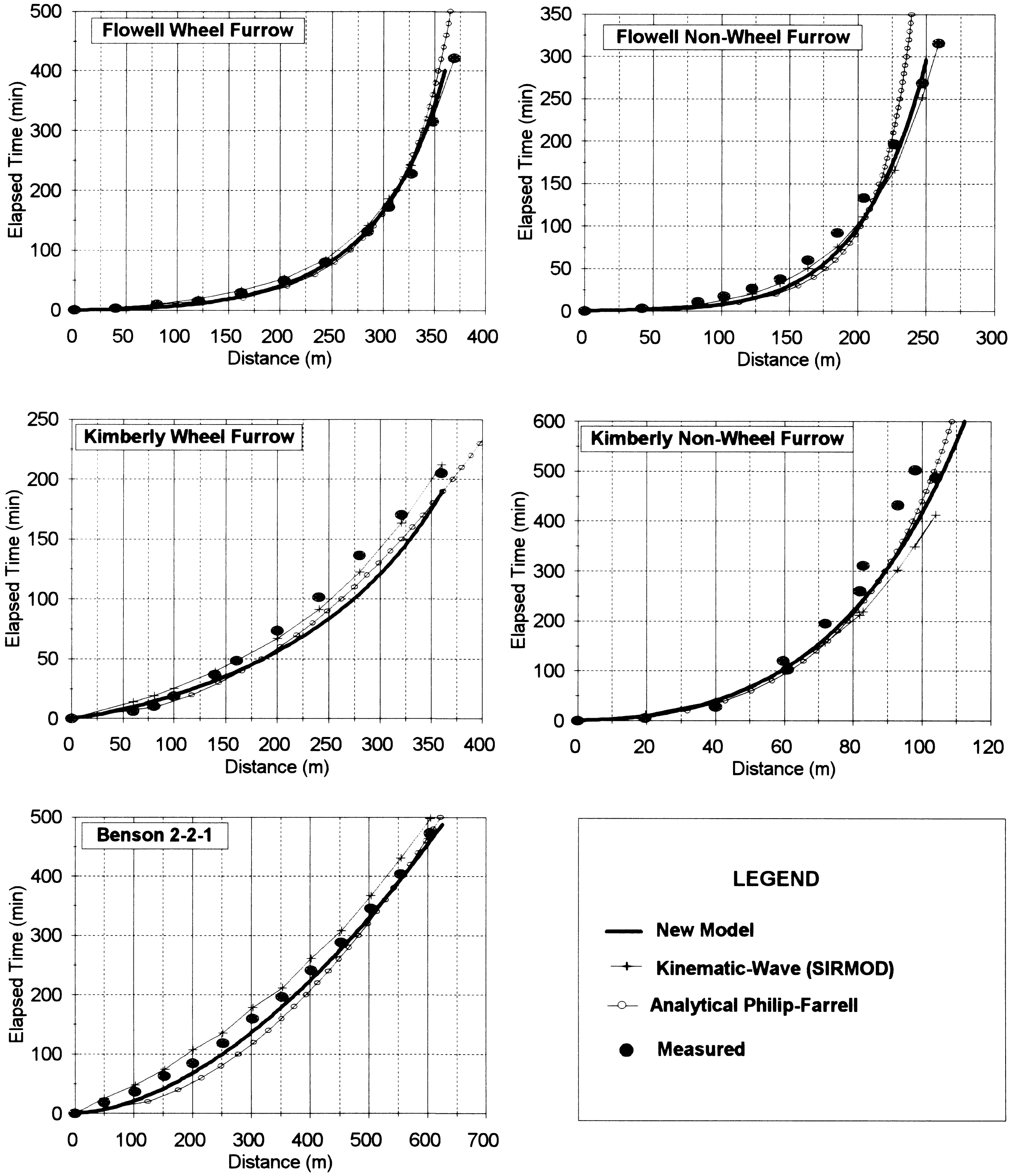


Fig. 3 Predicted and observed advance trajectories for five data sets (Walker and Humpherys 1983), kinematic-wave solution was obtained using SIRMOD (1989), and other prediction methods explained in the text

furrows. The results (Figs. 4 and 5) show reasonable agreement between measurements and predictions made by the proposed method. In some cases, the extension of the wetted area continued during recession, a phenomenon not accounted for by the proposed model which assumes instantaneous recession. In contrast, the kinematic wave model provides estimates for this phenomenon. To generalize the results depicted in Figs. 4 and 5, the empirical parameter updating scheme should be replaced by a physically-based one.

[1] Oliveira, C. A. S. 1989. Surge flow irrigation in a yellow-red Oxisol (internal report). CNPH/EMBRAPA, Brasilia, Brazil

Table 1 Input parameters for continuous flow case studies

Parameters	Flowell furrow		Kimberly furrow		Benson 2-2-1
	Wheel	Non wheel	Wheel	Non wheel	
Inflow rate (m^3/min)	0.120	0.120	0.090	0.048	0.068
Length (m)	360	250	360	360	625
Modified Kostiakov-Lewis:					
k ($m^3/m/min^a$)	0.002800	0.002170	0.008840	0.007010	0.018000
a	0.534	0.673	0.212	0.533	0.020
fo (m^3/m/min)	0.000220	0.000222	0.000170	0.000170	0.000100
Kostiakov-Lewis:					
k ($m^3/m/min^a$)	0.003060	0.004570	0.004930	0.006230	0.011500
a	0.625	0.597	0.459	0.600	0.206
Philip:					
S ($m^2/min^{1/2}$)	0.003000	0.002890	0.004370	0.007490	0.003130
A (m^2)	0.000247	0.000412	1.0E-08$^+$	0.000236	1.0E-08$^+$

$^+$ An arbitrary lower limit

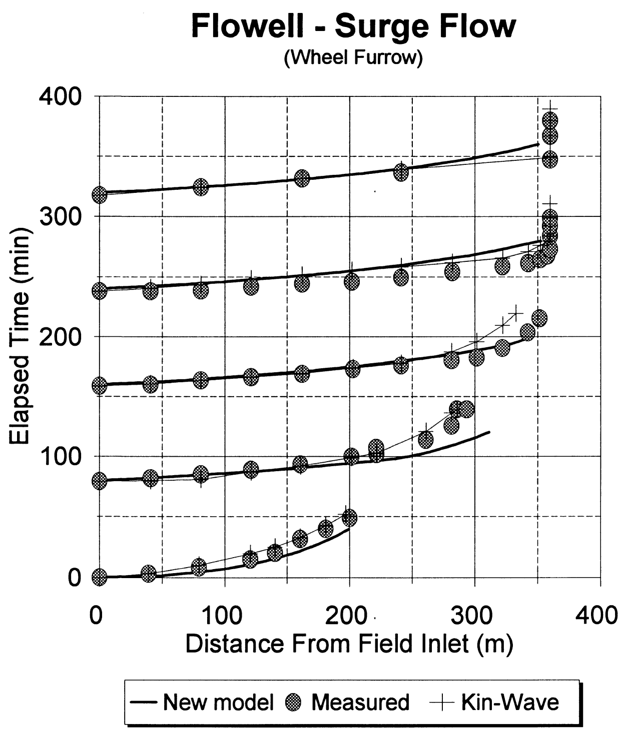


Fig. 4 Measured and predicted advance trajectories for 80-min cycled surge flow, measured data (full circles) and kinematic-wave solution (pluses) from Walker and Humpherys (1983), the proposed method is denoted by a thick line

Advance under spatially variable intake properties

In many fields, intake properties vary in space (Bautista and Wallender 1985) thereby limiting the applicability of models which assume constant intake along a furrow (Philip and Farrell 1964). The proposed method (Eq. 2) can accommodate spatial variations in intake properties along a furrow (Wallender 1986), or temporal variations in inflow rates. To illustrate advance trajectories in a spatially-variable field, we constructed a hypothetical 240 m furrow made up of four sections (each 60 m long) each with different intake properties. For the simulation, a furrow with intake data for wheel and non-wheel Kimberly (Table 1) were assigned to alternating sections (as indicated). Advance in the simulated furrow for a constant inflow rate $q = 0.12$ m^3/min is depicted in Fig. 6. The simulation results indicate faster advance rates on the "wheel" sections (as expected for low intake sections), and considerably-slower advance on the "non-wheel" section between 120 and 180 m. The advance was stopped completely at 120 m for nearly 30 min while infiltration rates on the wet portion of the furrow diminished to produce sufficient "runoff" (i. e., q_{in}-∫i[x,t-ts]) for further advance. Given detailed information on the spatial distribution of intake properties the method may be applied to an unlimited sequence of variations. Although this feature was not tested with field data, it is reasonable to expect a similar correspondence as found for surge flow tests with variable boundary conditions.

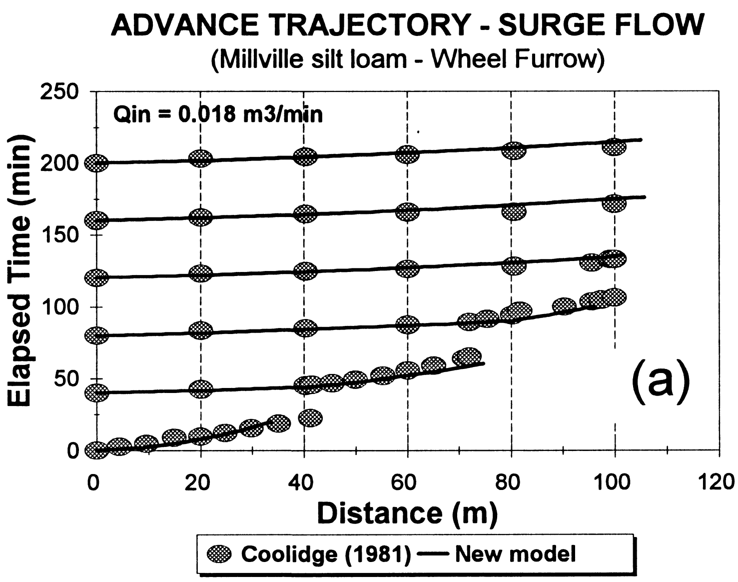
Interference of intake properties from advance information

Analytical solutions such as Philip and Farrell's (1964) offer a means for revealing relationships between the most important controlling variables, i. e., soil intake properties and inflow rate. Such solutions can form the basis for stochastic analyses of advance rates, considering infiltration parameters as random space functions (Jaynes and Clemmens 1986; Renault and Wallender 1994; Or and Walker 1996). Another practical application of analytical solutions is for solving the "inverse problem", i. e., inferring soil intake properties from advance rate information (Renault and Wallender 1991). To illustrate this point, we considered Flowell non-wheel furrow data (Walker and Humpherys 1983). A nonlinear regression procedure was used to fit the Philip-based analytical solution (Eq. 7) to the measured ad-

Table 2 Input parameters for surge flow case studies

Parameters	Flowell wheel		Millville wheel		Oxisol non wheel	
Inflow rate (m^3/min)	0.12		0.018		0.11142	
Length (m)	360		100		80	
	Dry	Wet	Dry	Wet	Dry	Wet
Modified Kostiakov-Lewis:						
k (m^3/m/min^a)	0.002800	0.004590				
a	0.534	0.356				
fo (m^3/$min^{1/2}$)	0.000220	0.000180				
Philip:						
S (m^2/$min^{1/2)}$			0.002800	0.001129	0.012683	0.008995
A (m^2)			1.0E-07$^+$	1.0E-07$^+$	0.002510	0.001189

$^+$ An arbitrary lower limit



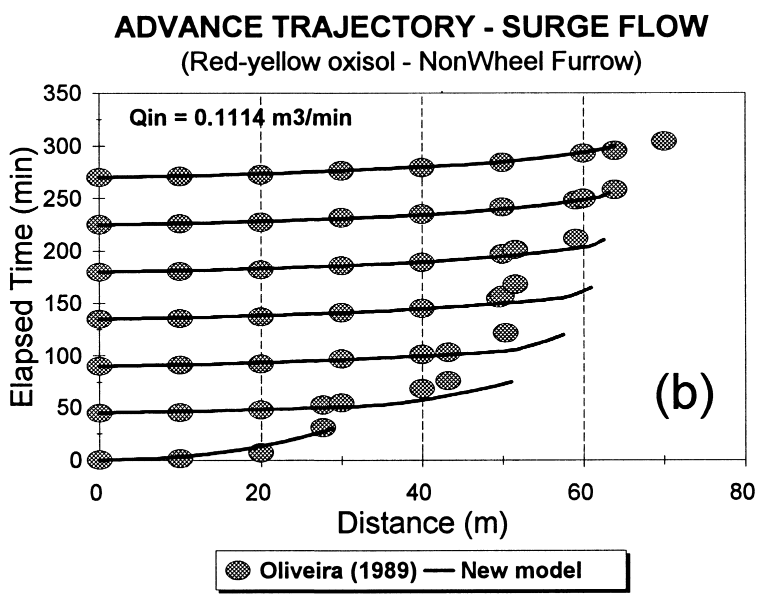


Fig. 5 a, b Measured and predicted advance trajectories for surge flow **a** in wheel furrow on Millville silt loam soil (80-min cycle time, ½ cycle ratio), and **b** in non-wheel furrow on Red-yellow oxisol (45-min cycle time, ⅔ cycle ratio); measured data (full circles) and the proposed method is denoted by a thick line

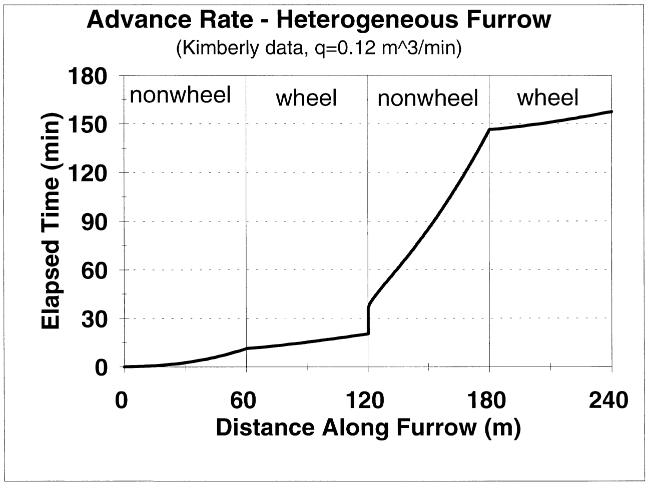


Fig. 6 Simulated advance in a furrow with variable intake properties data are based on Kimberly wheel and non-wheel furrow assigned to alternate 60 m sections

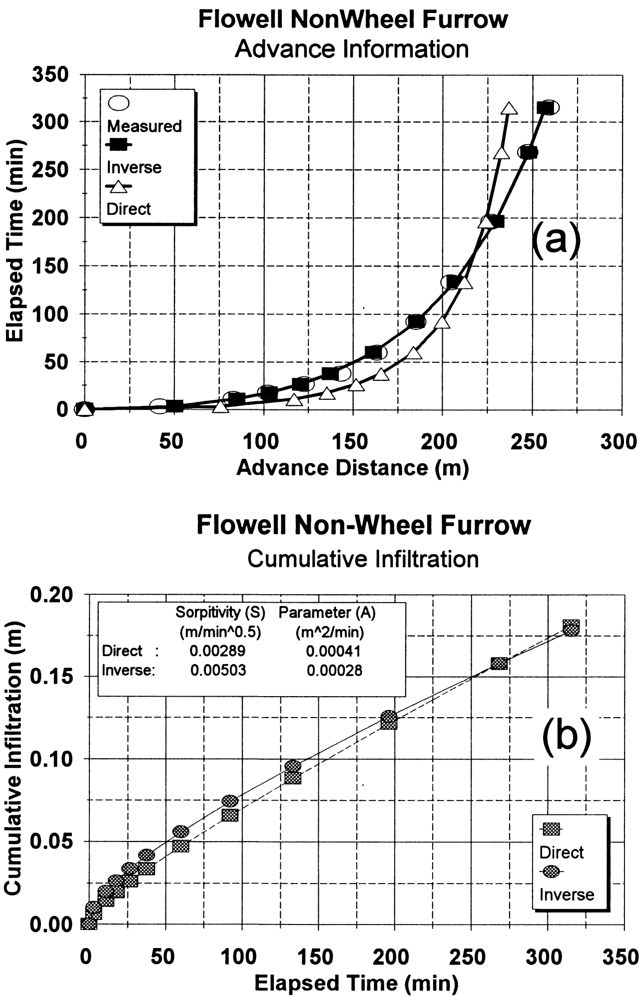


Fig. 7 **a** Estimated Philip infiltration parameters from Flowell non-wheel advance data by the "inverse" method (using Eq. 7). The "direct" predictions were also obtained by Eq. 7 using the reported infiltration parameters (Table 1). **b** A comparison of cumulative infiltration functions for Flowell non-wheel for "direct" and "inverse" parameters

vance trajectories. The resulting best-fit parameters S and A were denoted as "inverse" whereas original parameters (Table 1) were denoted as "direct". The excellent fit to observed advance rates is evident in Fig. 7a. Also included in Fig. 7a are predictions of advance trajectories made by using the "original" parameters in Eq. (7). A comparison of cumulative infiltration based on the two sets of parameters show little differences between the curves (Fig. 7b).

Summary and conclusions

A simple method for predicting advance rates in continuous and surge flow irrigation using only infiltration information was presented. The method was also tested under hypothetical conditions of spatially-variable intake properties. It may be viewed as a differential form of the Lewis-Milne equation using a fixed time step rather than a fixed spatial step, and ignoring surface storage. The reasonable predictability of advance trajectories based on infiltration information only emphasizes the critical influence of soil intake properties on the performance of surface irrigation systems. Simplified solutions, such as the proposed method, may be sufficient for solving practical problems, particularly when considering the large spatial variability in soil intake properties and in other parameters used by advanced numerical models.

Several analytical solutions with parameter requirements similar to the proposed method were re-evaluated. Analytical solutions offer practical tools for stochastic analyses of advance rates and evaluation of their impact on irrigation uniformity and efficiency under more realistic conditions than those usually considered in deterministic modeling. A simple application of analytical solutions for the inference of intake parameters from advance rate information (i.e., the "inverse problem") was presented.

The proposed method was tested successfully for intermittent water applications (surge flow). While surface storage seems to play a negligible role in determining advance rates under continuous flow, it becomes more important for surge flow conditions. A scheme for updating intake properties to account for changes in initial (or boundary) conditions was outlined for Philip's infiltration parameters, but was not tested due to lack of data.

In conclusion, the proposed method offers flexibility in handling spatial variations in soil properties along a furrow (or a basin) and can handle variations in inflow rate at the field's inlet. The intake and inflow parameters required by the method are simple and may be obtained by practitioners using either direct measurements (infiltrometers), or implementing "inverse" methods to infer intake from observed advance.

Acknowledgements The partial support of the Utah Agricultural Experimental Station (UAES-324), and a grant from the University New Faculty Grant Program (Utah Mineral Lease – AP79404) are gratefully acknowledged. We thank C. A. S. Oliveira (CNPH/EMBRAPA) for providing the red-yellow Oxisol data. We also thank CNPH/EMBRAPA and CNPq (Brazil) for providing partial support to the second author. The many useful comments made by four anonymous reviewers are gratefully acknowledged. Approved as UAES journal paper no. 4673.

References

Brooks RH, Corey AT (1964) Hydraulic properties of porous media. Hydrol Pap 3, 27 pp Colorado State University, Fort Collins

Bautista E, Wallender WW (1985) Spatial variability of infiltration in furrows. Trans ASAE 28: 1846 – 1851, 1855

Coolidge PS (1981) Advance rate under automated pulsed flow irrigation system. MS Thesis. Dept of Agricultural and Irrigation Engineering, Utah State Univ, Logan, Utah

Elliott RL (1980) Furrow irrigation field evaluation data. Department of Agricultural and Chemical Engineering, Colorado State University, Fort Collins, Co

Hall WA (1956) Estimating irrigation border flow. Agric Eng 37: 263 – 265

Jaynes DB, Clemmens AJ (1986) Accounting for spatially variable infiltration in border irrigation models. Water Resour Res 22: 1257 – 1262

Kostiakov AN (1932) On the dynamics of the coefficient of water percolation in soils and on the necessity for studying from a dynamic point of view for purposes of amelioration. Trans Sixth Comm Int Soil Sci Russian Part A, 17 – 21

Lewis MR, Milne WE (1938) Analysis of border irrigation. Agric Eng 19: 267 – 272

Lewis MR (1937) The rate of infiltration of water in irrigation practice. Trans Am Geophys Union 18th Annual meeting, 361 – 368

Or D, Walker WR (1996) Effects of spatially variable intake on surface irrigation advance. J Irrig Drain Eng ASCE 122: 122 – 130

Parlange J-Y (1977) A note on the use of infiltration equations. Soil Sci Soc Am J 41: 654 – 655

Parlange MB, Fuentes C, Haverkamp R, Parlange J-Y, Elrick D, Price MJ (1993) Optimal solutions of the Bruce and Klute equation. Soil Science 155: 1 – 7

Philip JR (1957 a) The theory of infiltration, 4. Sorptivity and algebraic infiltration equations. Soil Science 84: 257 – 264

Philip JR (1957 b) The theory of infiltration, 5. The influence of the initial moisture content. Soil Science 84: 329 – 339

Philip JR (1957 c) The theory of infiltration, 2. The profile at infinity. Soil Science 83: 435 – 448

Philip JR, Farrell DA (1964) General solution of the infiltration-advance problem in irrigation hydraulics. J Geophys Res 69: 621 – 631

Renault D, Wallender WW (1991) ALIVE advance linear velocity: Horton infiltration law from field water advance rate. Trans ASAE 34: 1706 – 1714

Renauld D, Wallender WW (1994) Furrow advance rate solution for stochastic infiltration properties. J Irrig Drain Eng ASCE 119: 74 – 90

Russo D, Bresler E (1980) Field determination of soil hydraulic properties for statistically analyses. Soil Sci Soc Am J 44: 697 – 702

Samani ZA, Yitayew M (1989) Changes in soil properties under intermittent water application. Irrig Sci 10: 177 – 182

Scheidegger AE (1957) The physics of flow through porous media. Mcmillan, New York

Shayya WH, Bralts VF, Segerlind LJ (1993) Kinematic-wave furrow irrigation analysis: a finite element approach. Trans ASAE 36: 1733 – 1742

Silva HR (1995) Wetting induced changes in near-surface soil physical properties affecting surface irrigation. Ph.D. dissertation, Dept. of Biological and Irrigation Engineering, Utah State University, Logan, Utah

SIRMOD (1989) The surface irrigation simulation model (SIRMOD) user's guide. Dept of Agricultural and Irrigation Engineering, Utah State University, Logan, Utah

Strelkoff T (1977) Algebraic computation of flow in border irrigation. J Irrig Drain Eng ASCE 109: 377 – 392

Swartzendruber D (1993) Revised attribute of the power form infiltration equation. Water Resour Res 29: 2455 – 2456

Walker WR, Skogerboe GV (1987) Surface irrigation theory and practice. Prentice-Hall, Inc, Englewood Cliffs, New Jersey

Walker WR, Humpherys AS (1983) Kinematic-wave furrow irrigation model. J Irrig Drain Eng ASCE 109: 377 – 392

Wallender WW (1986) Furrow model with spatially varying infiltration. Trans ASAE 29: 1012 – 1016